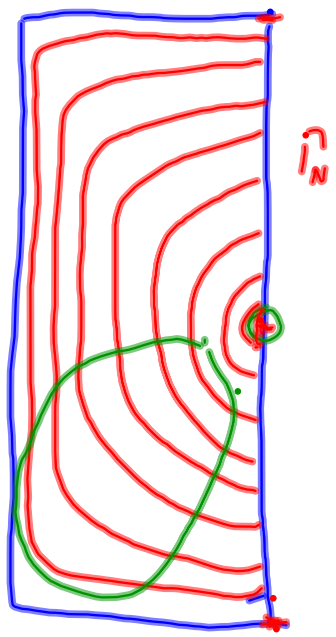
Wireless:

Guest Wireless
9709442331

- image segmentation
- anisotropic diffusion
- stokes in a long tube
- stochastic pdes/glaciers
- mg-dd
- additive mg
- parallel mg
- weighted-norm fosls
- spectral-polarmetric signal fitting
- cr
- time-space mg/parareal
- uq
- exascale
- local-schur non-galerkin mg
- parallel amr

$$\boxed{\nabla\, \underline{b}\_ \;\; = f}$$

$\langle A Q_v, Q_v \rangle \leq$

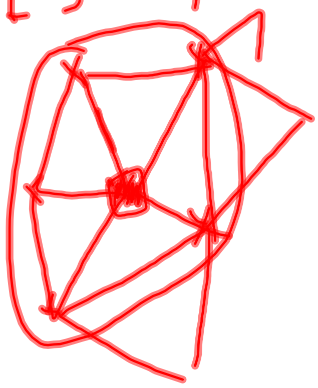


$PR = Q$

$RP = I$

$R = [0 \;\; I]$

$P: \begin{bmatrix} P_F \\ I \end{bmatrix}$ ← F-pts ← C-pts



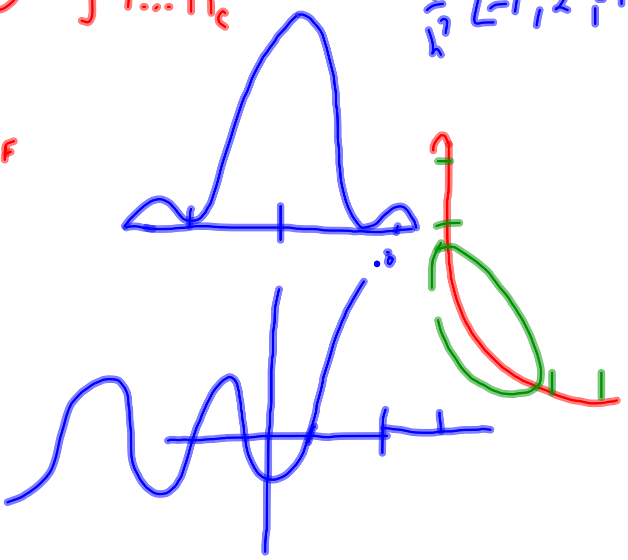
$P_F = (C^* P_{tent})\big|_F \qquad C^3 \qquad C A_{sssfr} \qquad C\begin{bmatrix}1&0&1\\1&1&0\\0&0&1\end{bmatrix}$

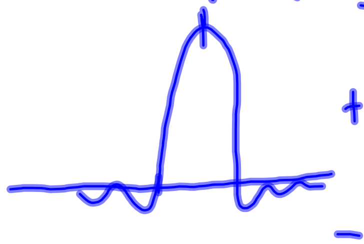
$AP_j = 0 \quad j = 1 \dots n_c$

$\frac{1}{h^2}[-1, 2, -1] \quad \mathbb{R}[\frac{2}{3}, \frac{1}{6}] \quad [1/3, 1]$

$P\mathbb{1}_c = \mathbb{1}_F$

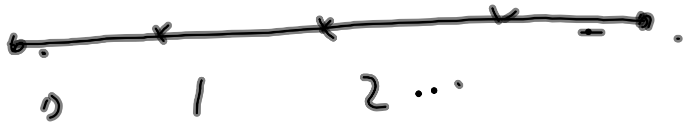
$P = \begin{bmatrix} P_F \\ I \end{bmatrix} \qquad P\begin{bmatrix} P_F & 1 \\ I & 0 \\ 0 & I \end{bmatrix}$





$L = \partial_x$

$p' = f$



$$\sum_{\Delta} \int_{\Delta} (p' - f)^2 \, dx$$

$$-p_{i-1} + 2p_i - p_{i+1} = h^2 \cdots$$

$$\left\langle L \sum_{i=1}^{n} \phi_i p_i , L \phi_j \right\rangle = \langle f, L\phi_j \rangle \quad j = 1..n$$

$$\sum_{i=0}^{n-1} (p_{i+1} - p_i)^2$$

$p_0 =$ given

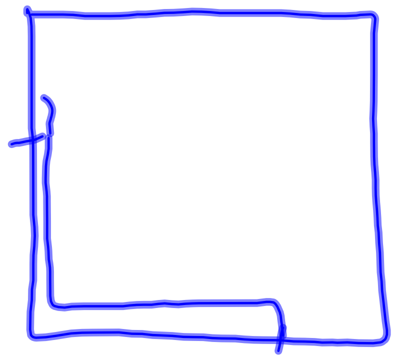
$\nabla \cdot \underline{F}(u) = 0$

$F(u_0)\cdot \nabla u$ + Stoff

$\underline{b}\cdot\nabla u + g(u) =$

$\langle \underline{b}\cdot\nabla u, v\rangle = \langle g(u), v\rangle$

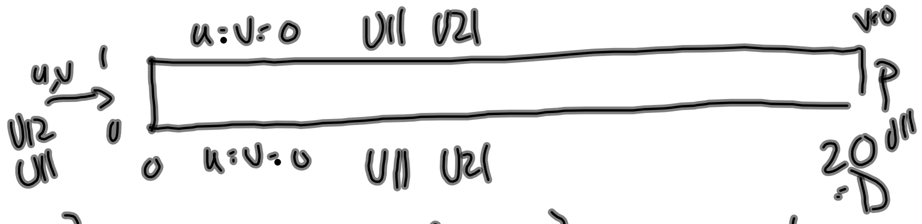
$\langle Lu, Lv\rangle = \langle g(u), Lv\rangle$



$\Gamma_{in} =$
$\{\underline{b}\cdot\underline{n} < 0\}$

$$-\Delta \underline{u} + \nabla p = f$$

$$\nabla \cdot \underline{u} = 0 \quad \delta p$$



$$\underline{U} = \nabla \underline{u} \qquad \begin{bmatrix} \underline{U11} & \underline{U21} \\ \underline{U12} & U22 \end{bmatrix}$$

$$\begin{cases} \nabla \times \underline{U} = 0 \\ -\nabla \cdot \underline{U} + \nabla p = f \end{cases}$$

$$\mathrm{tr}(\underline{U}) = 0$$

$$\begin{bmatrix} -2\Delta & 0 & 0 & -\partial_x^2 + \partial_y^2 \\ 0 & -\Delta & 0 & \partial_{xy} \\ 0 & 0 & -\Delta & \partial_{xy} \\ -\partial_x^2 + \partial_y^2 & \partial_{xy} & \partial_{xy} & -\Delta \end{bmatrix} \begin{bmatrix} U11 \\ U12 \\ U21 \\ p \end{bmatrix}$$

$$C_0 \sim \frac{1}{D^3}$$

$$\rho = .997 = 1 - \frac{1}{D^2}$$

$$\frac{2\pi h^2}{D^2}$$

$f(u)$ $U22 = -U11$

$$F(\underline{U}, p; f) = \|\nabla \times U\|^2 + \|-\nabla \cdot \underline{U} + \nabla p\|^2$$

$$c_0 \|u\|_1 \le F(u; 0) \le c_1 \|u_1\|_1$$

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$\lambda_{\max} \ll \lambda^{1}_{\min}, \lambda^{2}_{\min}$$

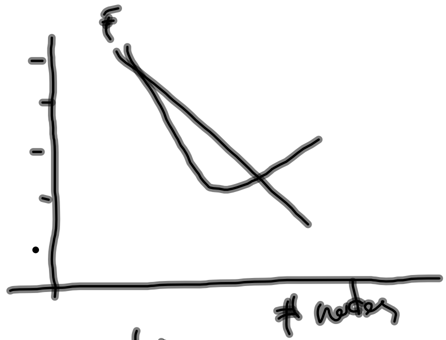
AMG + CG

e_1 $\quad r_{ij} = A_{ij} e_j$

e_2 $\quad \left| \sum_j r_{ij} \right|$ ~~small~~ $= |r_i|$

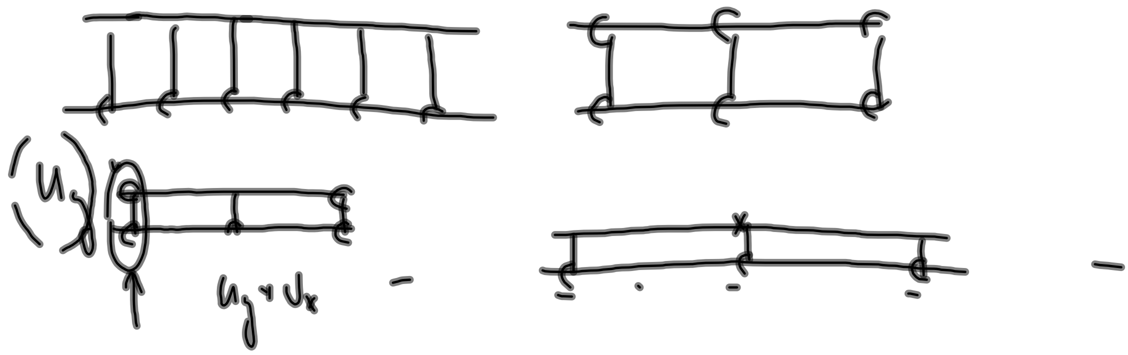
e_3

e_4



$\| \underline{U} - u^h \|$

$$\begin{bmatrix} .018 & 0 & 0 & .018 \\ 0 & .17 & 0 & .19 \\ 0 & 0 & .0013 & .0013 \\ .034 & .0091 & .0001 & .034 \end{bmatrix} \quad \begin{matrix} .33 \cdot 10^{-4} \\ .42 \cdot 10^{-5} \\ .61 \cdot 10^{-5} \\ .21 \cdot 10^{-4} \end{matrix}$$


u_y
$u_y + v_x$

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \cdot e$$

h^{2c}

$A_{11} e_1$

$A_{22} e_2$

$$\frac{1}{D^2}|u|_1 \le \|Lu\|^2$$

$e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$

$D = \frac{1}{h^2}$ $\quad (.997)^m = .85$

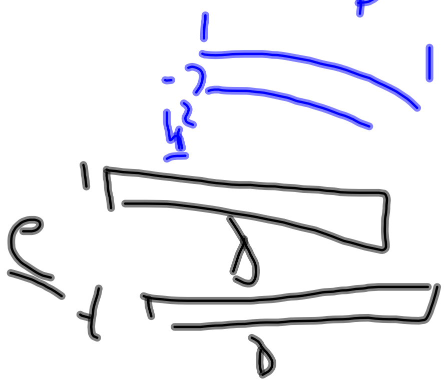
$(.003)m = .15$

500

$\frac{h^2}{D^2}$

e_1, e_2

$\cancel{\frac{h^4}{D^2}}\, h^{\varepsilon}$ $\quad \frac{t^2 h^2}{D^2}$



Change Detection

- Differencing

→ Noisy

One Solution

Step 1: Segment

Pan

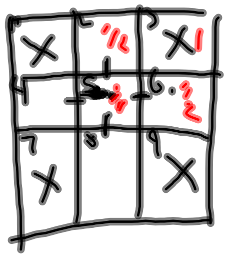
NIR
R
G
B

$\}$ [0, 2047]

SWA

$$A_{ij} \; (M^{[0]}+M^{[1]}) = \begin{cases} e^{-\alpha |I_i^{[0]} - I_j^{[1]}|} & \text{for } i,j \text{ in 4-hood} \\ 0 & \end{cases}$$

$(Pe_i)^T A Pe_i \quad \frac{10}{4}$



$$L_{ij} \; (M^{[0]}+M^{[1]}) = \begin{cases} -A_{ij} & \text{if } i \neq j \\ \sum_k A_{ik} & \text{if } i = j \end{cases}$$

L_{ii}/G_{ii}

$\Gamma_{ij} = A_{ij}/V_{ij}$

1/2	5/4	1
1/4	1	5/4
0	1/4	1/2

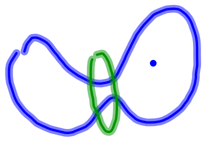
$$V_{ij}^{[w]} = \begin{cases} 0 & \text{if } A_{ij} = 0 \\ 1 & \text{if } A_{ij} \neq 0 \end{cases}$$

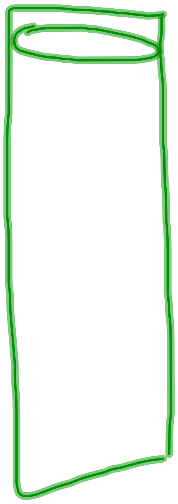
$$G_{ij} = \begin{cases} -V_{ij} & \text{if } i \neq j \\ \sum_{k \neq i} V_{ik} & \text{if } i = j \end{cases}$$

$$\Gamma_i^{i[r+1]} = \frac{L_{ii}/G_{ii}}{A_{ii}/V_{ii}}$$

$$a_{ii}^{r} = a_{ii}^{t} + \sum_{j\neq r} w_{ji} a_{ji} + \sum_{j\neq i} a_{ij} w_{ji} + \sum_{j,k\neq i} w_{ji} a_{jk} a_{ki}$$

$$M^{[r+1]} = \frac{M^{[r]}}{\text{return} \bigcup_{M^{[r]}\times M^{[r]}}}$$





$$U^{[r]} = P^{[r,r+1]} U^{[r+1]}$$

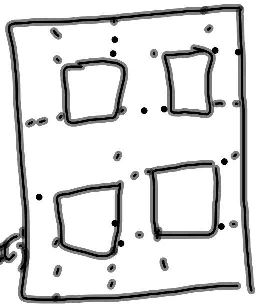
Recursive Part

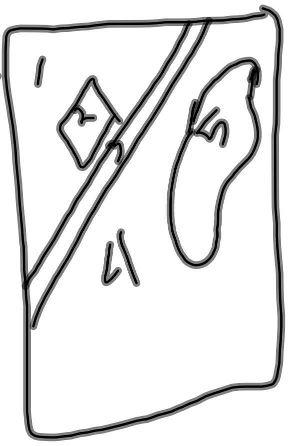
Call AMGcycle

(1) Coarsen A.

$$\rightarrow P^{[r,r+1]} = \begin{cases} 1 & i \in C \\ 0 & i \in C,\ j \neq i \\ \dfrac{A_{ij}}{\sum_{k \in C} A_{ik}} & i \notin C \end{cases}$$

$$\tilde{P}^{[r,r+1]}$$





(2) $I^{[r+1]} = (\tilde{P})^T I^{[r]}$

(3) $A^{[r+1]} = P^T A^{[r]} P$

$$\boxed{A_{ij} = A_{ij} e^{\alpha |I_i - I_j|}}$$

(4) $V^{[r+1]}, L^{[r+1]}, G^{[r+1]}$

$L = D - A$

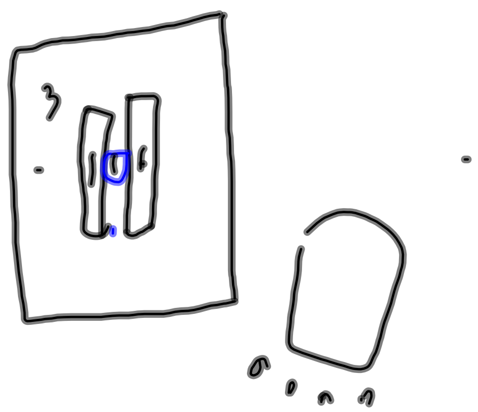
$P^T D P - P^T A P$

Summary

1. Build graph
2. Coarsen via AMG
3. Identify salient segments

Questions

1. No spatial connectivity of segments!



Parallel MG

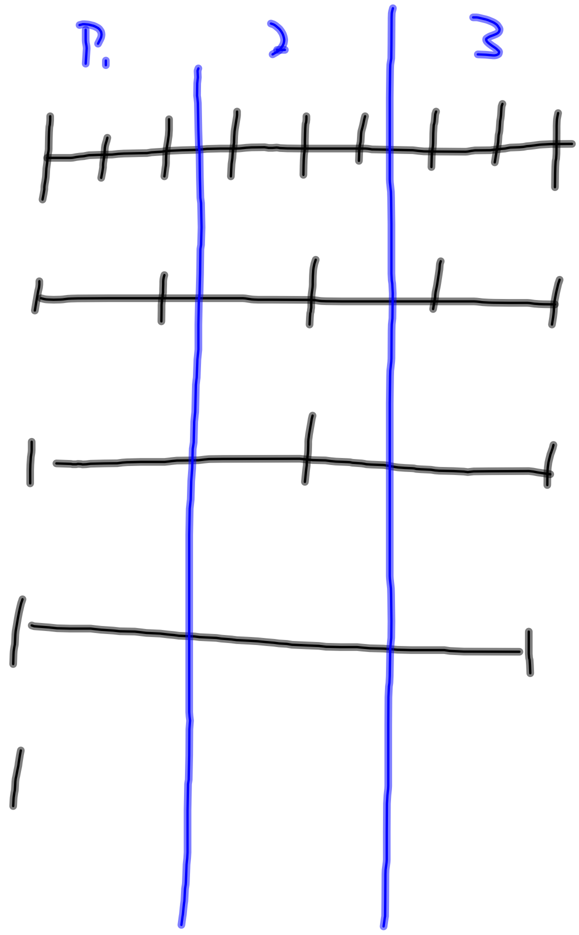
$T_{comm} = \alpha + m\beta$

α - latency

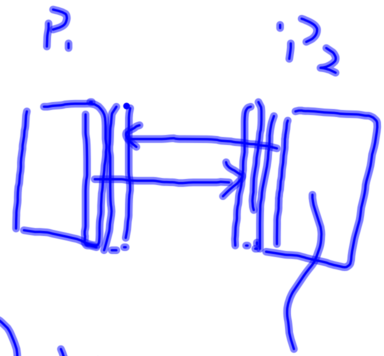
β - time for 1 double

m - # doubles

$T_{comp} = \gamma m$



Relaxation (2D)



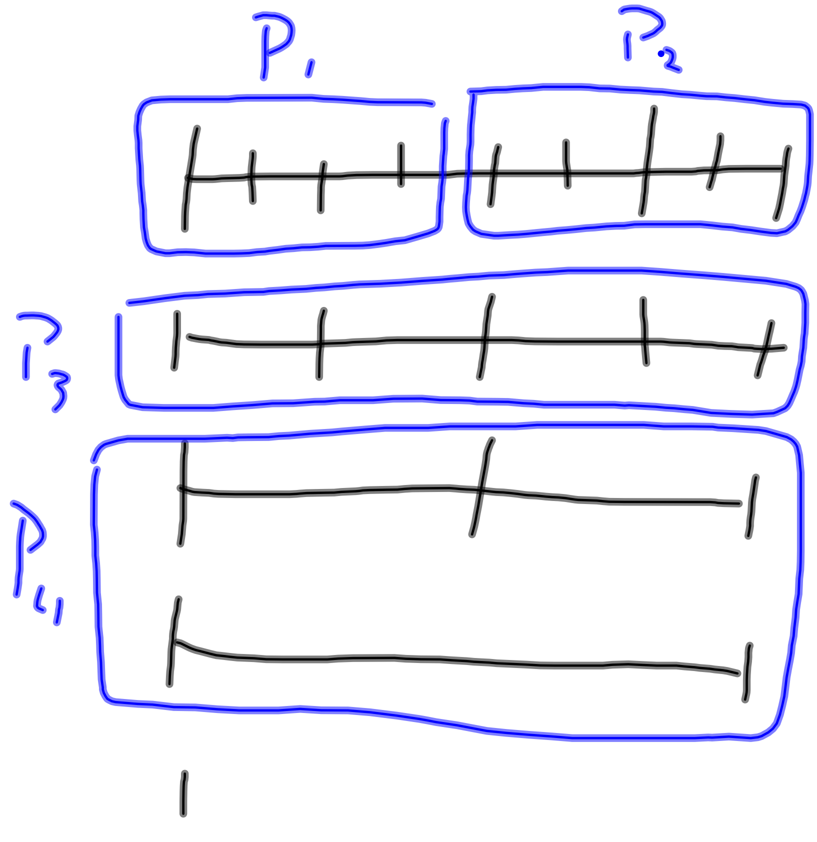
Ω

$\Gamma = \partial\Omega$

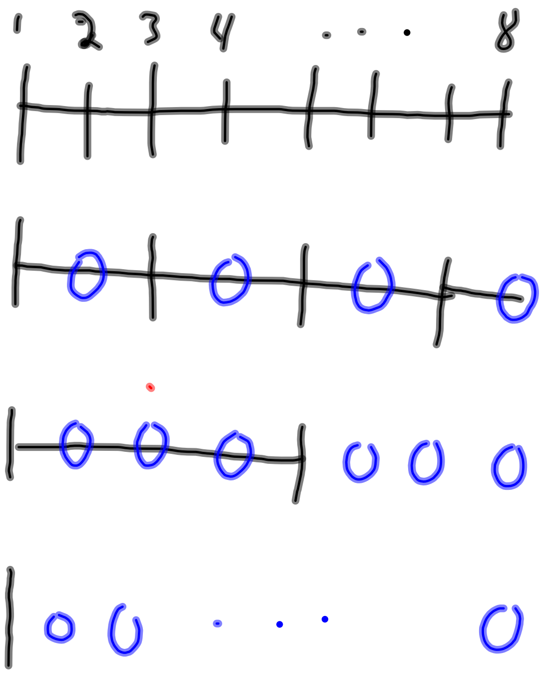
$$T_{rel} = 4\alpha + \Gamma\beta + 5\Omega\gamma$$

$$T_{MG} = \overbrace{(1+1+\cdots)}^{\log N}4\alpha + \left(1+\tfrac{1}{2}+\tfrac{1}{4}+\cdots\right)\Gamma\beta + \left(1+\tfrac{1}{4}+\cdots\right)5\Omega\gamma$$

$$= (\log N)\alpha + (2)\Gamma\beta + (4/3)5\Omega\gamma$$


P1
P2
P3
P4,1

$f \stackrel{=8}{=}$ coarsening factor



$F = 8/7$ grid comp.

Algorith = CN

work potential

$= \log_f(P)P - (FP - P)$

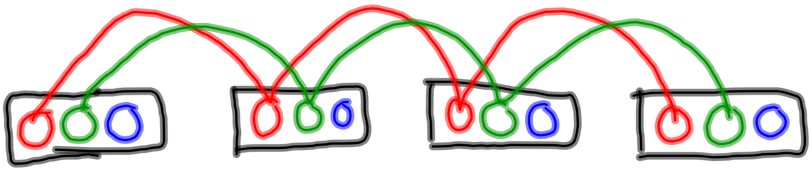
Best speedup $\leq \max\left\{1 + \frac{\log P}{n}, \frac{C}{F}\right\}$

$$\frac{FN + \log(P)P - FP + P}{FN}$$

$$\leq 1 + \frac{\log P}{n}$$

$$A_k = P_k^T A_{k-1} P_k$$

Parallel AMG



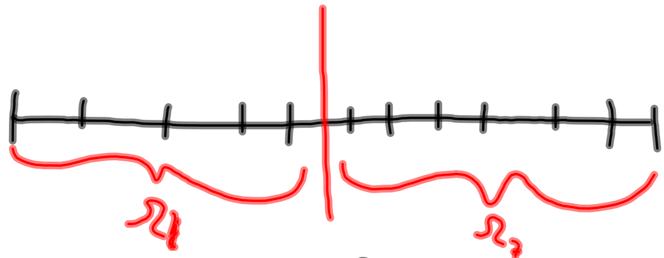
$\Delta_h u = f$ in Ω

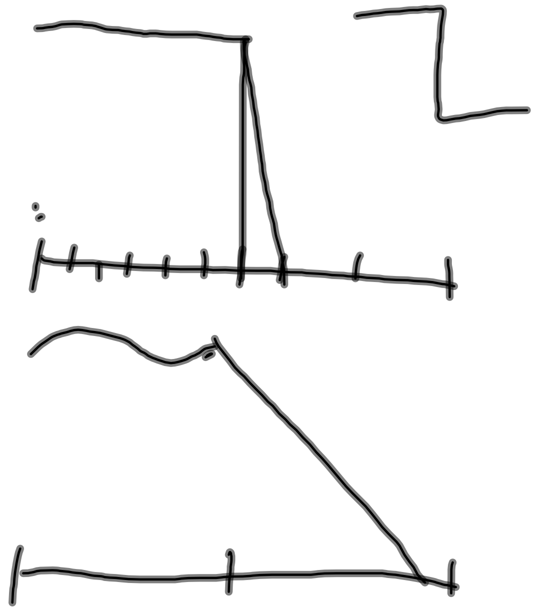
$u = 0$ in $\partial\Omega$

$u = \sum u_i$ $\quad f = \sum f\phi_i$ $\quad 1 = \sum \phi_i$

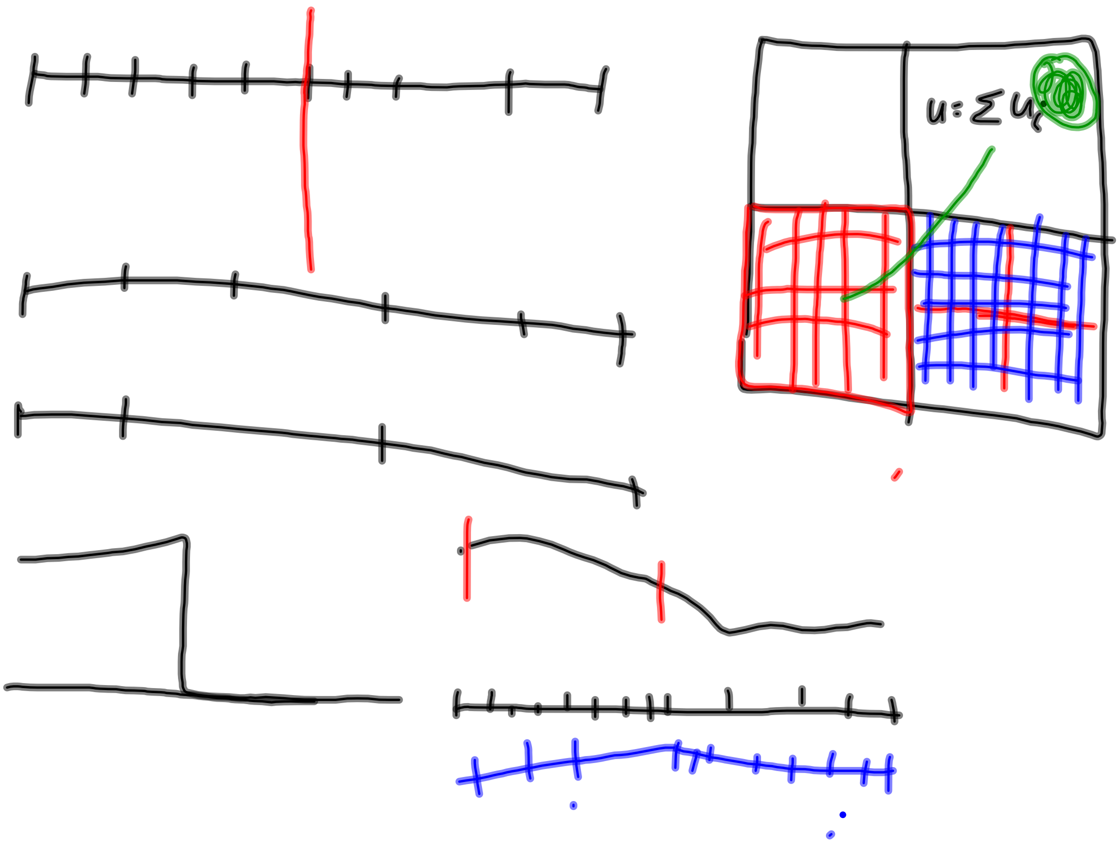
f

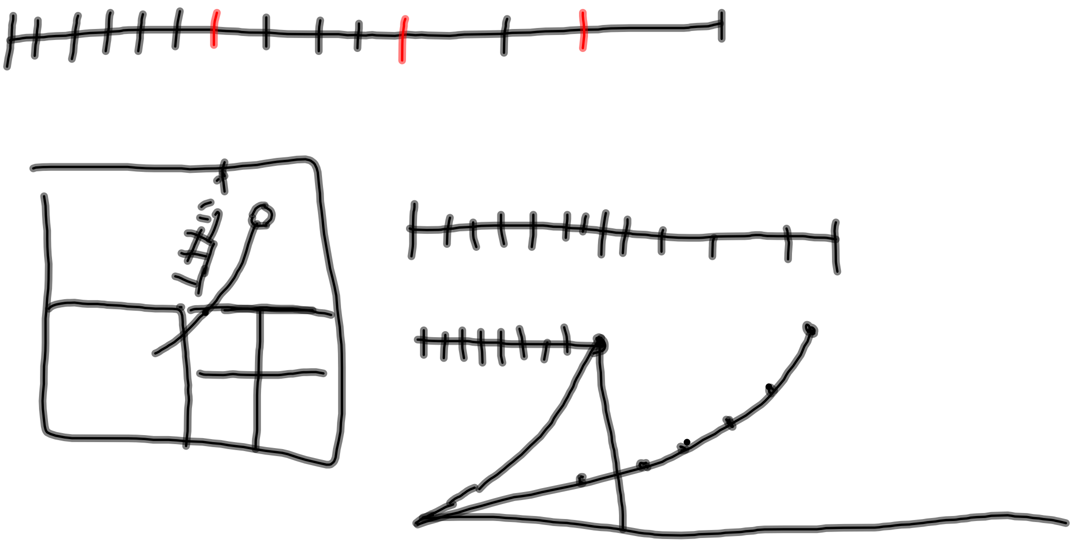
$f = \sum f_i$





$$\Delta_h u = \begin{cases} f & \text{in } \Omega_1 \\ 0 & \text{in } \Omega_2 \end{cases}$$


u=Σuᵢ



1) Build Problems

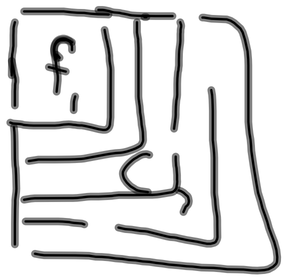
. Setup Coarse

$$Au = f$$

2) Solve

$$P^{-1}AP$$

$$u = \begin{bmatrix} u^f \\ u^c \end{bmatrix}$$



$$P^c = \begin{bmatrix} I & 0 \\ 0 & P_{22} \end{bmatrix}$$

After iteration

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$$

$$\sqrt{2}\left(\frac{\sqrt{K}-1}{\sqrt{K}+1}\right)^{\ell}$$

$$K = \operatorname{cond}_C(M^{-1}A)$$

17^2

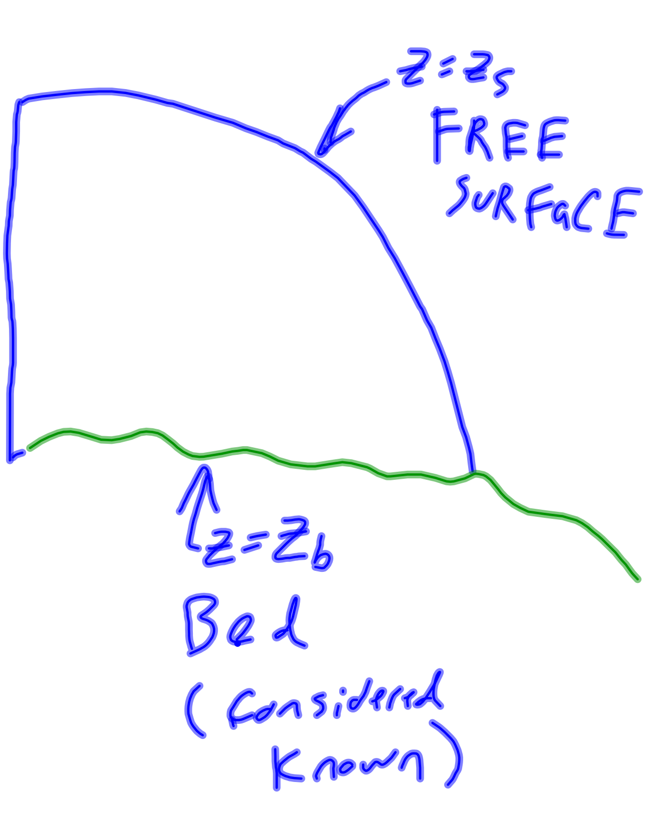
1×9

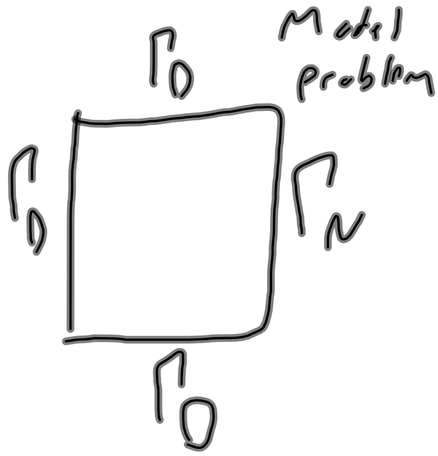
$$9 \cdot (3 \cdot 17)^2 = 9 \cdot 17^2 \cdot 9$$

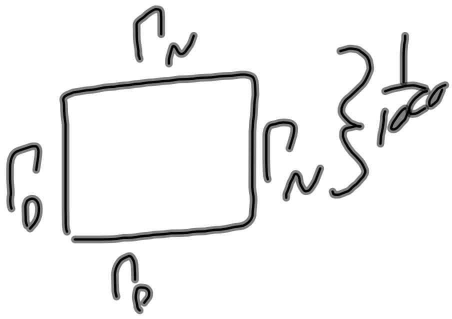
1.61

~~GS~~ V-cycle: 0.63

W-cycle: 0.51


z = z_s
FREE SURFACE
z = z_b
Bed
(considered known)


Model problem
Γ_D
Γ_D
Γ_N
Γ_D


Γ_N
Γ_D
Γ_N
Γ_D
1/1000

$\nabla \cdot \vec{u} = 0$ where $\vec{u} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$

$\sigma_{xy} \Rightarrow \begin{pmatrix} & * & \\ & & \\ & & \end{pmatrix}$

$$\cancel{\rho\left[\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u}\right]} = \nabla \cdot \overleftrightarrow{\sigma} + \rho \vec{g}$$

small

$\sigma_{kk} = (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$

$\sigma'_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}$

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = 0$$

$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = 0$$

$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = \rho g$$

$\sigma'_{ij} = 2\eta \dot{\varepsilon}_{ij}$

$\eta = \frac{1}{2}(A(\theta))^{-\frac{1}{n}} (\dot{\varepsilon}_e)^{\frac{1}{n}-1}$

$$2\dot{\varepsilon}_e^2 = \dot{\varepsilon}_{xx}^2 + \dot{\varepsilon}_{yy}^2 + \dot{\varepsilon}_{zz}^2 + 2(\dot{\varepsilon}_{xy}^2 + \dot{\varepsilon}_{xz}^2 + \dot{\varepsilon}_{yz}^2)$$

$$\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

$$\sigma_{zz} = -\rho g \left(z_s(x,y) - z \right)$$

$$\tilde{\vec{u}} = \begin{pmatrix} u \\ v \end{pmatrix}$$

$$\Gamma_D : \tilde{\vec{u}} = 0$$

$$\nabla \cdot A \nabla \vec{u} = \vec{f}$$

$$\Gamma_N : \vec{n} \cdot \left(A \nabla \vec{u} \right) = 0$$

$$\vec{f} = \begin{pmatrix} \rho g \frac{\partial z_s}{\partial x} \\ \rho g \frac{\partial z_s}{\partial y} \end{pmatrix}$$

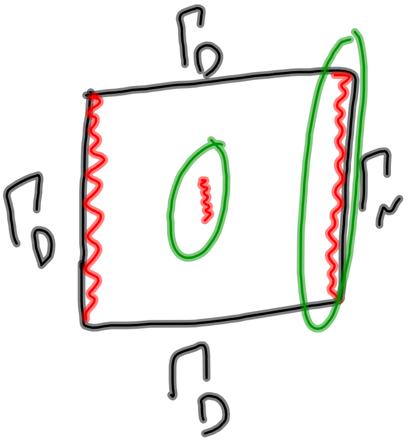
$$A = \eta \begin{pmatrix} 4 & 0 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\eta \approx \frac{1}{\left(\sum_i \sum_j (u_{i,j})^2 + \varepsilon_0 \right)^{1/3}}$$

$$\varepsilon_0 \approx 10^{-30}$$

$$w(x,y,z) - w(x,y,z_b)$$

$$= \int_{z_b(x,y)}^{z} -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) dz$$



$[0,1]^2$

$$u = v = \sin(\pi y)\sin^2(\pi x)$$

$$\tilde{A} = \eta \begin{pmatrix} 4 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 2 & 0 & 0 & 4 \end{pmatrix}$$

Size	V	W
128	.12	.09
512	.20	.11
1024	.21	.11

2D (x, y)

$$-\nabla \cdot D \nabla p + \underline{b} \cdot \nabla p = f$$

Polynomial chaos

$$D = \begin{pmatrix} d_{11} & 0 \\ 0 & d_{22} \end{pmatrix} \quad d_{11}(\xi_1, \eta_1) \quad d_{22}(\xi_2, \eta_2)$$

$$b_2 = \sum_{j=0}^{5} \beta_j \xi_5^{j}$$

$$\underline{b} = \begin{pmatrix} b_1(\xi_3, \eta_3) \\ b_2(\xi_5) \end{pmatrix}$$

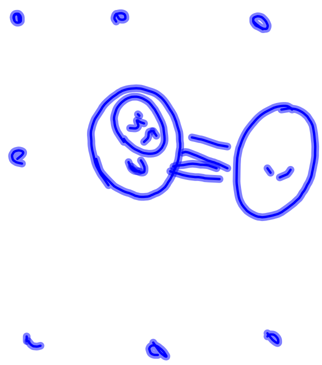
$T_{\xi_5 \xi_4 \xi_3 \xi}$ ⊗ PDF

$$\begin{pmatrix} -\beta_1 \Delta & \cdot \beta_2 \Delta \\ \cdot \beta_3 \Delta & \cdot \beta_4 \Delta \end{pmatrix}$$

$$(T \otimes P_{de})^{-1} = T^{-1} \otimes P_{de}^{-1} \begin{pmatrix} \cdot & & 0 \\ & \cdot & \\ & & \cdot \end{pmatrix} \underline{P}_{35}$$

$T(x,y)$

Nodal Based AMG (MG)



$$\begin{pmatrix} -\Delta & \partial_{xy} \\ \partial_{xy} & -\Delta \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

$$\begin{pmatrix} \boxed{A_{11}} & A_{12} \\ A_{21} & \boxed{A_{22}} \end{pmatrix}$$

Unknown Approach

$$\begin{bmatrix} \square & \square & & \square \\ \square & \square & & \\ & & \square & \ddots \end{bmatrix} \begin{array}{l} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{array}$$

Nodal App

CR methods

primary $\quad I - (S^T M S)^{-1}(S^T A S) \quad$ F-relax.

habituated $\quad S^T(I - M^{-1}A)S$

$$\|V(1,1,P_*)\|_A \leq C\rho_{CR} \quad \text{not sharp}$$

$$P_* = \begin{bmatrix} -A_{ff}^{-1}A_{fc} \\ I \end{bmatrix}$$

Δ - symmetry

ω -

CR - compatible relaxation

$I - M^{-1}A$ smoother

P interp

$R = [0, I]$ C-pt

$S = \begin{bmatrix} I \\ 0 \end{bmatrix}$ F-pt

$RS = 0$

PR, $RP = I$

$P = \begin{bmatrix} W \\ I \end{bmatrix}$

Sharp CR:

$$R^T = \begin{bmatrix} 1/4 & \frac{1}{2} & 1/4 \\ \frac{1}{2} & 1 & \frac{1}{2} \\ 1/4 & \frac{1}{2} & 1/4 \end{bmatrix}$$

1st idea $R^T = \begin{bmatrix} W_r \\ I \end{bmatrix} \Rightarrow S = \begin{bmatrix} I \\ -W_r^T \end{bmatrix}$

$$(I - M^{-T}A)(I - \pi(R^T))(I - M^{-1}A)$$

2-grid : .181

CR(1) : .187

(2) : .183

$$\bar{K}^T = P_* = \begin{bmatrix} -A_{ff}^{-1} A_{fc} \\ I \end{bmatrix} \qquad S = \begin{bmatrix} A_{ff} \\ A_{cf} \end{bmatrix}$$

$$I - (S^T M S)^{-1} (S^T A S)$$

$$\| V(1,1,P_*) \|_A = \lambda_{max} \left[(I - M^{-1} A) \underbrace{S}_{S = \begin{bmatrix} I \\ 0 \end{bmatrix}} (S^T A S)^{-1} S^T A \, (I - M^{-1} A) \right]$$

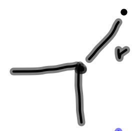
$$M_{CF} = \begin{bmatrix} M_{ff} & A_{fc} \\ 0 & M_{cc} \end{bmatrix}$$

$$\| v(1,1,P_*) \|_A \leq \rho_{CR}^2$$

$$G = \|Lu - f\|^2 = \|\nabla\cdot u\|^2 + \|\nabla\times u\|^2$$

$$G_w = \|w(Lu-f)\|^2 = \|w\nabla\cdot u\|^2 + \|w\nabla\times u\|^2$$

$w = r^{?}$



$\alpha_k = \frac{2k}{3}$

$$L^* L = (\quad) \begin{pmatrix} \nabla\cdot \\ \nabla\times \end{pmatrix} = \begin{pmatrix} -\Delta & 0 \\ 0 & -\Delta \end{pmatrix}$$

$$L^* w^2 L = \cdots = w^2 \begin{pmatrix} -\Delta & 0 \\ 0 & -\Delta \end{pmatrix} + \begin{pmatrix} -(\nabla w^2)\cdot\nabla & (\nabla^\perp w^2)\cdot\nabla \\ -(\nabla^\perp w^2)\cdot\nabla & -(\nabla w^2)\cdot\nabla \end{pmatrix}$$

Problem:

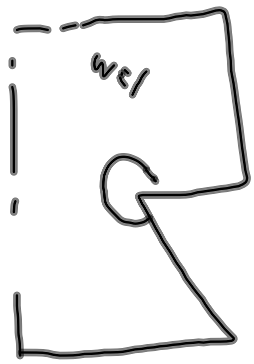
$$w_1(L w_2 \underset{\text{smooth}}{\phi} - f)$$

$w_2\phi = \underline{u}$

$p \sim r^\alpha \sin(\alpha\theta)$

$\alpha = 2/3,$

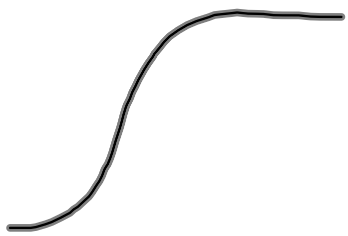
$\underline{u} = \nabla p$

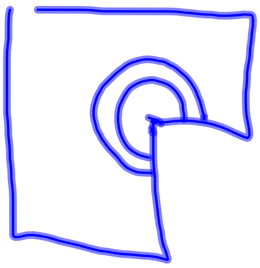


$$\|\underset{\sim}{w}\nabla u_1\|^2 + \|w\nabla u_2\|^2 + \|r^{\beta-1}\underline{u}\|^2$$

$$-\left(\nabla\cdot w^2\nabla u_1 + \nabla\cdot w^2\nabla u_2\right)$$

?





$\frac{3\pi}{2}$

$$p = r^{2/3}\sin(2/3\,\theta) = \phi$$

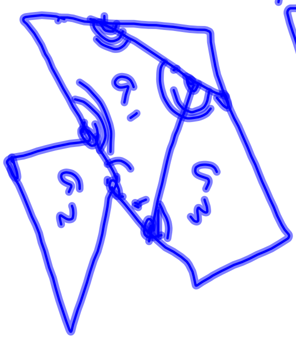
$$r^{\frac{2k}{3}}\sin\left(\frac{2k}{3}\theta\right)$$

$-\Delta p = f$

$\delta(r)\phi$

$p = 0$

$-\nabla\cdot a\nabla$

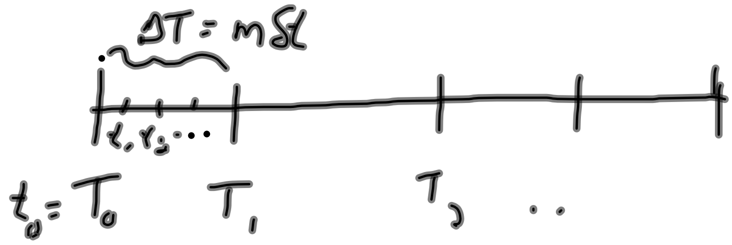


$u = u_0 + (u_1)$

$u_0 = \alpha\phi$

Parareal - Lions, Maday, Turincini (2001)
Gander, Vandewalle
Barry Lee ...
Michael Minion

Parallel time integration



linear time propagators F_δ, F_Δ

Want to solve $A_\delta u_\delta = g_\delta = \begin{bmatrix} u(0) \\ 0 \\ \vdots \\ 0 \end{bmatrix}$

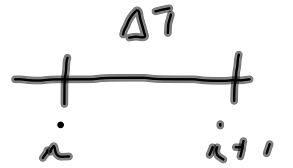
$$A_\delta = \begin{bmatrix} I & & & \\ -F_\delta & I & & \\ & & \ddots & \\ & & -F_\delta & I \end{bmatrix}$$

Pararea l solves coarse system

$$A_\Delta U_\Delta = g_\Delta$$

$$A_\Delta = \begin{bmatrix} I & & & \\ -F_\delta^m & I & & \\ & & \ddots & \\ & & -F_\delta^m & I \end{bmatrix}$$

Parareal Algorithm:



Initial guess U^0_Δ ~ let this be serial coarse F_Δ

For $k = 0, 1, \ldots$

Distribute U^k_Δ to processors

Compute in parallel $F_\delta^m U^k_{\Delta,i}$

Communicate results to single processor

Compute in serial

$$U^{k+1}_{\Delta,i+1} = F_\Delta U^{k+1}_{\Delta,i} + F_\delta^m U^k_{\Delta,i} - F_\Delta U^k_{\Delta,i}$$

Parareal is just

$$u_\Delta^{k+1} = u_\Delta^k + B_\Delta^{-1}\left(g_\Delta - A_\Delta u_\Delta^k\right)$$

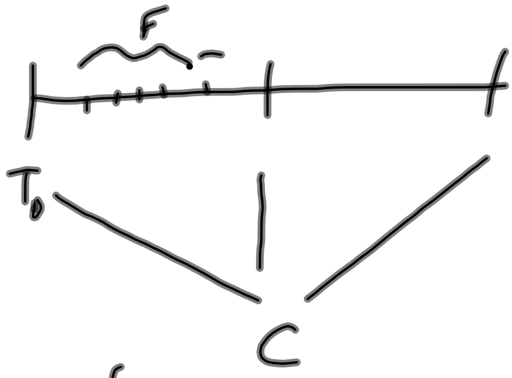
$$B_\Delta = \begin{bmatrix} I & & \\ -F_\Delta & I & \\ & & \ddots \end{bmatrix}$$

$$= B_\Delta^{-1}\left(B_\Delta u_\Delta^k + g_\Delta - A_\Delta u_\Delta^k\right)$$

u(0)

$$= B_\Delta^{-1}\begin{bmatrix} F_\delta^m u_{\Delta,1}^k - F_\Delta u_{\Delta,1}^k \\ \vdots \end{bmatrix}$$

$$A = \begin{bmatrix} A_{ff} & A_{fc} \\ A_{cf} & A_{cc} \end{bmatrix}$$



$$R_F = \begin{bmatrix} -A_{cf} A_{ff}^{-1} \; , & I_c \end{bmatrix}$$

$$S = \begin{bmatrix} I \\ 0 \end{bmatrix}$$

$$P_F = \begin{bmatrix} -A_{ff}^{-1} A_{fc} \\ I \end{bmatrix}$$

$$0 = \left(I - S(S^T A S)^{-1} S^T A\right)\left(I - P_F \underbrace{(R_F A P_F)}_{A_\Delta}{}^{-1} R_F A\right)$$

Coarsest Grid

=> distribute procs.

solve PDE

Est. Error

Refinement.

Load Balance.

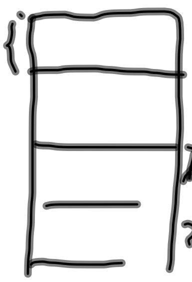
$Ax = b$.

AMG setup.

Poisson. 10k proc

<- 10%

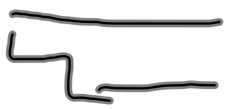
5%



10% (Ax = b)

70% (Refine Load)

Navier Stokes



90%

5% <-

$\rho \to 1.0$

Johannes Kraus $\rightarrow$ $B_c = \sum$ local Schur comps.

$$\frac{1}{4} S \le B_c \le S$$

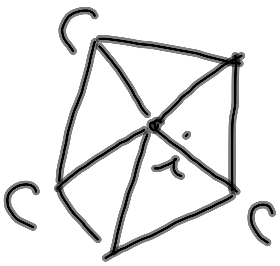
$$S_{cc} = A_{cc} - A_{cf} A_{ff}^{-1} A_{fc} \qquad A = \begin{bmatrix} A_{ff} & A_{fc} \\ A_{cf} & A_{cc} \end{bmatrix}$$

$$= P_*^T A P_*$$

$$P_* = \begin{bmatrix} -A_{ff}^{-1} A_{fc} \\ I \end{bmatrix} \rightarrow \underset{P=\begin{bmatrix} W \\ I \end{bmatrix}}{\min}\ \underset{e_r}{\max} \ \arg\min \frac{\|(I - PR)e\|_2}{\|e\|_A}$$

AMGe $\longrightarrow A = \sum_{\varepsilon} A_\varepsilon$

Construct P

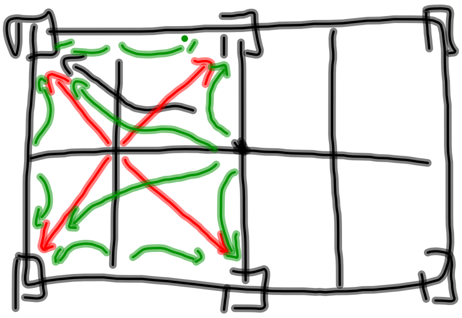


Ω_i

localize weak a. prop.

$$\frac{\|E_i^T(I-PR)e\|_2}{\|e\|_{A_i}}, \quad A_i = \sum_{\varepsilon\in\Omega_i} A_\varepsilon$$

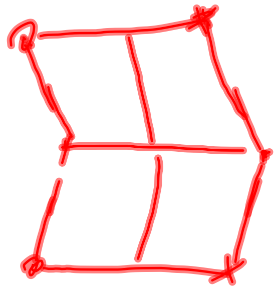
Problem w. AMGe

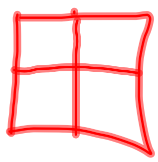


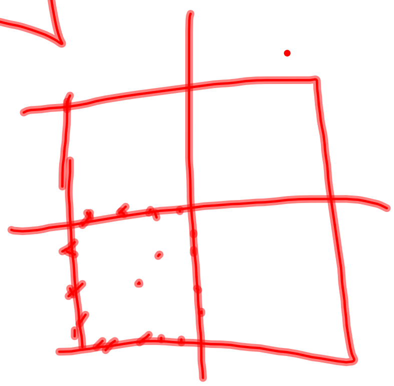
Need coarse elts.
& coarse stiffness mats.

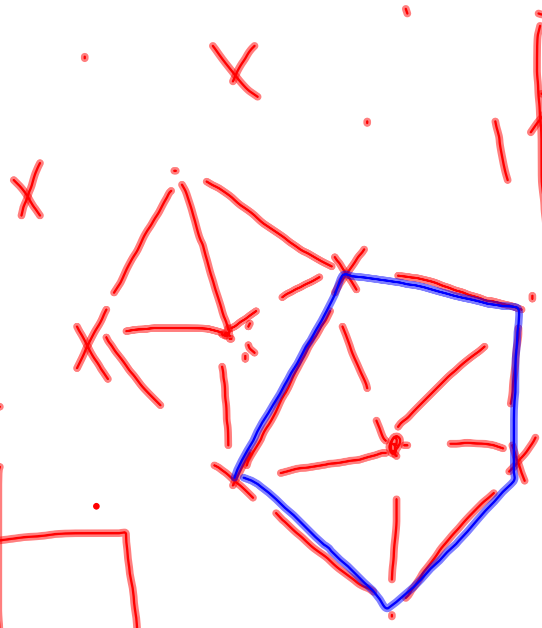
$$\sum_i \overset{*}{P}_i^T A_i P_i^* \approx P^{*T} A P^* ?$$

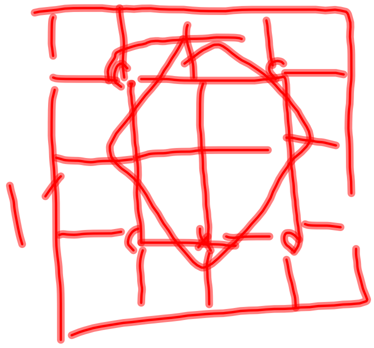
i = coarse elts











$$\min_{e = e_\Gamma^*} \langle Ae, e \rangle = \langle S e_b, e_b \rangle$$