Network: Guest Wireless

Password: 970 944 2331

AMG for H(div)

Trzaskio

ADS, AMS, BoomerAMG

→ Hybridization

$-\Delta p + p = \operatorname{div} f$

$(u,v) + (\operatorname{div} u, \operatorname{div} v) = (f,v)$

$u, v \in RT$

$p = \operatorname{div} u$

$$\begin{cases} (u,v) + (p, \operatorname{div} v) = (f,v) \\ (\operatorname{div} u, q) - (p,q) = 0. \end{cases}$$

$$\begin{bmatrix} M & B^T \\ B & -W \end{bmatrix} \begin{bmatrix} u \\ p \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix}$$

← hybridization

→ $[u \cdot n] = 0$

$\int_F [u \cdot n] \mu \, ds = 0$

$$\begin{bmatrix} A & B & C \\ B & -W & 0 \\ C & 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ p \\ \mu \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} M & B^T \\ B & W \end{bmatrix}^{-1} \begin{bmatrix} C \\ 0 \end{bmatrix}}$$

s.p.d.

~ elliptic eqn.

$$\left\| \begin{matrix} u - \nabla p = 0 \\ \nabla \cdot u \quad = f \end{matrix} \right\|^2 \xrightarrow{\text{ADS}} \begin{pmatrix} I - \nabla \nabla \cdot & -\nabla \\ \nabla \cdot & -\Delta \end{pmatrix}$$

Hypre.

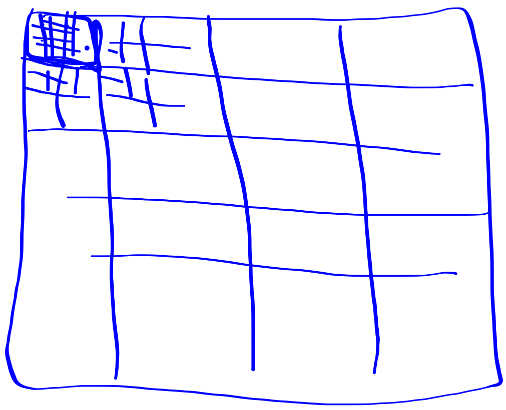
$-\varepsilon \Delta - \nabla \nabla \cdot$

$-\varepsilon \nabla \times \nabla \times - (1+\varepsilon) \nabla \nabla \cdot$

AMG-DD

(Domain Decomp.)

$Au = f$



On each proc:

$$A_c^p u_c^p = f_c^p \quad (c = \text{composite})$$

Final soln:

$$u = \sum_p Q_p u_c^p$$

FAMG: $O(\log(p)^2)$

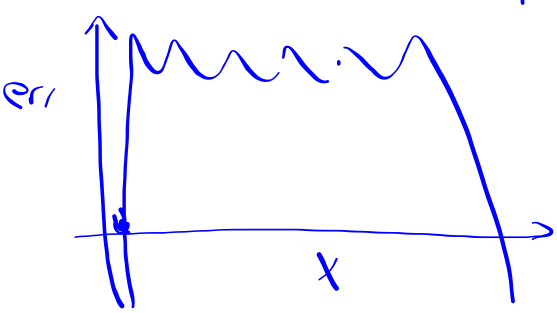
AMG-DD: $O(\log(p))$

comm cost

Interpolation:

$$\|u^h - P u^{2h}\|$$

A has row sum zero in the interior



WAP $\dfrac{\|u - Pv\|^2}{\|u\|^2} \le \dfrac{c}{\|A\|} \dfrac{\langle Au, u\rangle}{\langle u, u\rangle}$

SAP $\|u - Pv\|_A^2 \le \dfrac{c}{\|A\|} \langle Au, Au\rangle$

$\dfrac{\|A\|}{c} \|u - Pv - P\bar{w}\|^2$

$\le \|u - Pv\|_A^2 \le \dfrac{c}{\|A\|} \langle A^T A u, u\rangle$

$$\nabla \cdot \left(\mu\left(\nabla \underline{u} + (\nabla \underline{u})^T\right)\right) - \nabla p = f$$

$$\nabla \cdot \underline{u} = 0$$

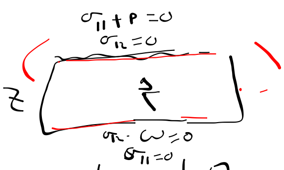
$$\mu = \frac{1}{2} A(T)^{-\frac{1}{n}} \dot{\varepsilon}_e^{\frac{1}{n}-1} + c$$

$$\dot{\varepsilon}_e = \sqrt{\frac{1}{2} \sum_{ij} |\nabla \underline{u} + \nabla \underline{u}^T|^2}$$

$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & -\sigma_{11} \end{bmatrix} = \nabla u + \nabla u^T$$

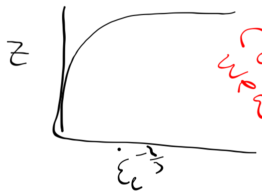
$$\phi = \frac{1}{\mu}$$

$$\begin{cases} \phi \hat{\sigma} \sim \nabla u \\ \nabla \cdot \hat{\sigma} - \nabla p = f \\ \nabla \cdot u = 0 \\ \frac{1}{\phi + c} \nabla \times \phi \hat{\sigma} = 0 \end{cases}$$

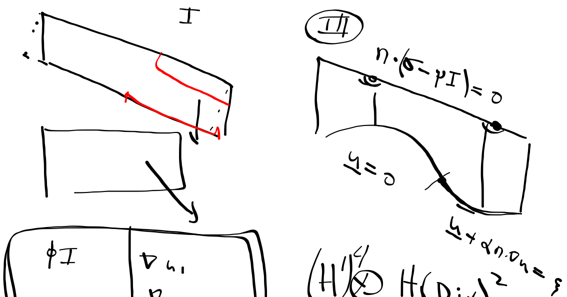


$\phi = 1.0$

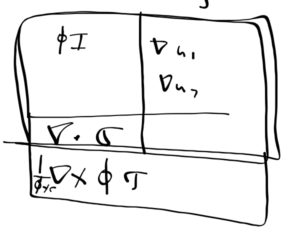
$\phi =$ exact



$\phi \to 0$ Top
Complete BC
weak



$$\frac{(H^1)^4 \otimes H(\mathrm{Div})^2}{(H^1)^6}$$



$$\begin{bmatrix} \phi & & & \\ & \phi & & \\ & & \phi & \\ & & & 0 \end{bmatrix} \quad \begin{matrix} -\partial x + \partial y \\ -\partial y - \partial x \\ -\partial y + \partial x \\ -\partial x - \partial y \end{matrix}$$

$$\nu(\nabla u + \nabla u^T) \qquad O(|u|^{\frac{1}{3}})$$

$$\begin{bmatrix} A_{ff} & A_{fc} \\ A_{cf} & A_{cc} \end{bmatrix} \qquad A_{ff} e_f + A_{fc} e_c = 0 \longrightarrow \begin{bmatrix} e_f \\ e_c \end{bmatrix} \quad \begin{bmatrix} W e_c \\ e_c \end{bmatrix}$$

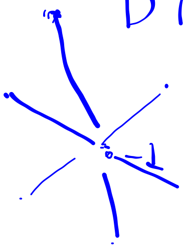
$$e_f = -A_{ff}^{-1} A_{fc} e_c$$

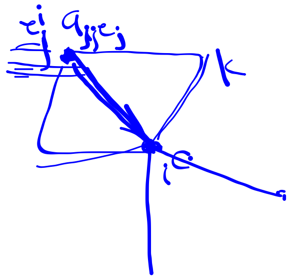
$$R = P^T \qquad e_f = P e_c \qquad R = \begin{bmatrix} -A_{cf} A_{ff}^{-1} & I \end{bmatrix}$$

$$A^c = P^T A P \qquad \underline{R = -A_{cf} A_{ff}^{-1}} \qquad P = \begin{bmatrix} W \\ I \end{bmatrix}$$

$$A^T d = 0$$

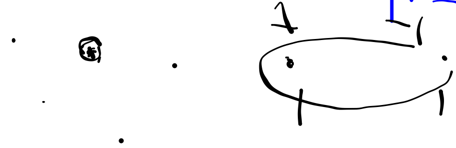
$$DA \qquad D^{-1} A$$





$$\sum_k x_{ik} a_{kj} = 0 \qquad \forall j \in N^R(i)$$

$$-r^T A e_i = 0$$



$$E_{TG} = (I - \Pi_A)(I - M^{-1}A), \quad \Pi_A = P(P^TAP)^{-1}P^TA$$

$$\|E_{TG}(P)\|_A^2 = 1 - \frac{1}{\sup_v K(v)} = \lambda_{n_c+1}, \quad K(v) = \frac{\|(I - \Pi_{\tilde M})v\|_{\tilde M}^2}{\|v\|_A^2}$$

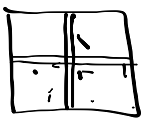
$$P_* = \begin{bmatrix} -A_{ff}^{-1}A_{fc} \\ I \end{bmatrix}, \quad \tilde M = M(M + M^T - A)^{-1}M^T \qquad |C| = n_c$$

$$-2cI + L + U$$

$$Ax = \lambda \tilde M x, \quad x_1, \dots, x_n, \quad P_0 = [x_1, \dots, x_{n_c}]$$

$$\bar P_0 = \begin{pmatrix} P_f \\ P_c \end{pmatrix}, \quad \underline{\underline{\hat P_0 = \begin{bmatrix} P_f \\ P_c \end{bmatrix} P_c^{-1}}} \underset{F\text{-relax}}{=} P_* \qquad P^TAP_x = \lambda P^T \underset{\tilde M}{\ } P_x$$

$$\begin{bmatrix} \varepsilon & 1 \\ 1 & \varepsilon \end{bmatrix}$$



MILU.

$$A = D - L - U$$

$$M = (\Delta - L)\Delta^{-1}(\Delta - U)$$

$$M^{-1}A$$

$$\begin{cases} -\Delta u = f & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

discretization: linear. Lagrange.

Find u_h. $a(u_h, v) = f(v) \quad \forall v \in V_h$.

iterate $\bar{u}_h$

$\|u_h - \bar{u}_h\|_A = ?$

We can estimate.

Estimate.

$c\eta \le \|u - u_h\|_A \le \eta$.

$\|u - \bar{u}_h\|_A = ?$

$\|u - \bar{u}_h\|_A^2 = 2J(\bar{u}_h) - 2J(u)$

$$\|u - u_h\|_A \le \sup_{v \in H^1} \frac{a(u - u_h, v - v_I)}{\|v\|_{H^1(\Omega)}}$$

$a(u - u_h, v) = 0 \quad \forall v \in V_h$.

$J(u) = \min_v J(v)$

$J(v) = \frac{1}{2} a(v, v) - f(v)$

$\|u - \bar{u}_h\|_A^2 = 2J(\bar{u}_h) - 2J(u) = \underline{2J(\bar{u}_h) - 2J^*(\vec{\sigma})}$

$J^*(\vec{\sigma}) = \max_{\vec{\tau}} J^*(\vec{\tau}) = J(u)$

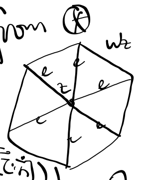
$J^*(\vec{\tau}) = -\frac{1}{2}(\vec{\tau}, \vec{\tau}) \quad \tau \in H(\mathrm{div}), \ \underline{\mathrm{div}\,\vec{\tau} = f}$

$\|u - \bar{u}_h\|_A^2 = 2J(\bar{u}_h) - 2J^*(\vec{\sigma}) \le 2J(\bar{u}_h) - 2J^*(\vec{\sigma}_h)$

How to get $\vec{\sigma}_h$

Solve a local hybridized mixed problem ⊛

local $\eta \le$ use the stability estimate from ⊛



$\vec{\sigma}^\Delta := \vec{\sigma} - (-\nabla \bar{u}_h)$

local $\vec{\sigma}^\Delta$
$$\begin{cases} (\vec{\sigma}^\Delta, \vec{\tau})_{\omega_z} - (g, \nabla\cdot\vec{\tau})_{\omega_z} - (\lambda, [\vec{\tau}\cdot\vec{n}]) = 0 \\ (\nabla\cdot\vec{\sigma}^\Delta, \mu)_{\omega_z} = (f, \mu)_{\omega_z} + \underline{\text{correction}} \\ ([\vec{\sigma}^\Delta\cdot\vec{n}], c)_e = ([\nabla\bar{u}_h\cdot\vec{n}], c)_e. \end{cases}$$

local $\eta = \|\text{local } \vec{\sigma}^\Delta\|$.

$0 = a(u - u_h, v) \quad \forall v \in V_h$.

$a(u - \bar{u}_h, v) = a(\overline{u_h - \bar{u}_h}, v) \quad \forall v \in V_h$

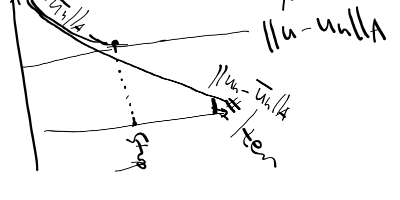
$\underline{f(v) - \sum_e \int [\nabla\bar{u}_h\cdot\vec{n}]\, v}$ ($= a(u - \bar{u}_h, v)$)

$$\text{local } \eta \le \left(\|u - \bar{u}_h\|^2_{\text{local } A} + \|\bar{u}_h - u_h\|^2_{\text{local } A} \right)$$

$\|u - \bar{u}_h\|_A \le \eta$.

$\|B^{-1} r\|$.

$\kappa(B^{-1}A)$



$\sqrt{A^TA}$: $Q = UV^T$

$Q^TA = VU^TU\Sigma V^T = V\Sigma V^T$

$AQ^T = U\Sigma V^TVU^T = U\Sigma U^T$

$Ax = b$

$AQ^Ty = b \quad x = Q^Ty$

$Q^TAx = Q^Tb$

Properties of $L = D - A$ (symmetric) $Lx = b$

- sparse
- scale-free degree dist.
- "small-world" property

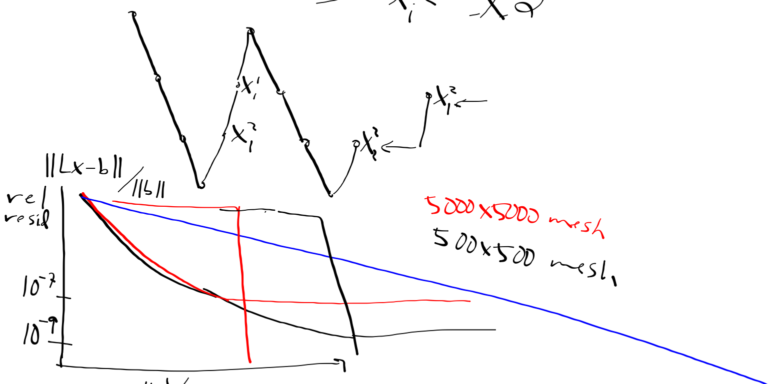
LAMG

- unsmoothed agg.
- elimination of low-degree
- energy inflation min. agg.
- recombination

$Lx = b$

$X = \begin{bmatrix} | & & | \\ x_{i-r} & \cdots & x_i \\ | & & | \end{bmatrix}$ $\alpha = \operatorname{argmin}_{\beta} \|LX\beta - b\|_2$

$x_i \leftarrow X\alpha$



Assume we have P

solving $x = \operatorname{argmin}_y E_{tot}(y)$

$E_{tot}(x) = \frac{1}{2}x^T L x - x^T b$

coarse grid correction

$\tilde{x} \leftarrow \tilde{x} + Pe^c$

$e^c = \operatorname{argmin}_{y^c} E_{tot}(\tilde{x} + Py^c)$

Solve

$L^c e^c = b^c \quad L^c = P^T L P \quad b^c = P^T(b - L\tilde{x})$

assume error e

an ideal P satisfies

$PP^T e = e$

Reality: correction contaminated by energy inflation factor

$q(e) = \dfrac{E(PP^Te)}{E(e)} \rightarrow e^c \approx \frac{1}{q(e)} P^T e$

$\text{ACF} \approx 1 - \frac{1}{q(e)}$

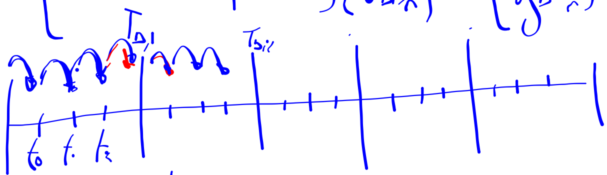
$E(x) = \sum_u E_u(x) \qquad E_u(x) := \sum_{v \neq u} a_{uv}(x_u - x_v)^2$

$u_t = f(\underline{x}, t) \qquad u(t=0) = u_0$

$\underline{u}_{k+1} = \Phi(u_k, u_{k+1}) + g(\underline{x}, t) \qquad k = 0, 1, 2, \dots, N_t$

$$A u = \begin{bmatrix} I & & & & \\ -\phi & I & & & \\ & -\phi & I & & \\ & & \ddots & \ddots & \\ & & & -\phi & I \end{bmatrix} \begin{bmatrix} u_0 \\ u_1 \\ \vdots \\ u_N \end{bmatrix} = \begin{bmatrix} g_0 \\ \vdots \\ g_N \end{bmatrix}$$

$$A_\Delta = \begin{bmatrix} I & & & \\ -\phi^m & I & & \\ & -\phi^m & \ddots & \\ & & \phi^m & I \end{bmatrix} \begin{bmatrix} u_{\Delta,1} \\ \vdots \\ u_{\Delta,\frac{N}{n}} \end{bmatrix} = \begin{bmatrix} g_{\Delta,1} \\ \vdots \\ g_{\Delta,\frac{N}{n}} \end{bmatrix}$$



Replace $\phi^m \to \Phi_\Delta$

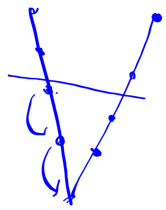
$u_{t+1} = u_t + \Delta t\, f(u_{t+1})$

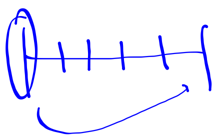
$\Phi \sim \left(I - \frac{\Delta t}{\Delta x^2} A\right)$

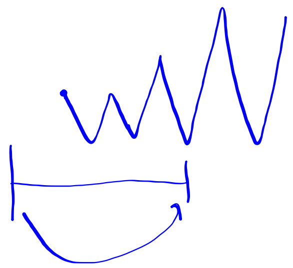
① Spatial Coarsening

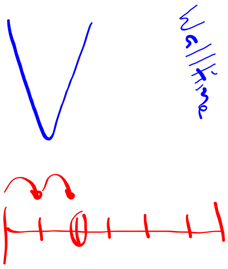
② Improved initial guess

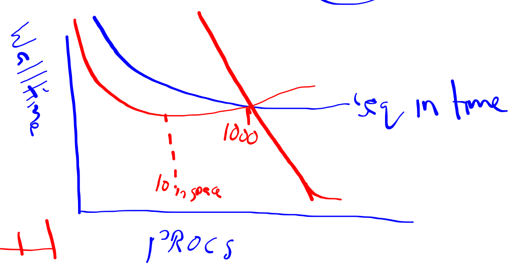
① Spat Coarsen



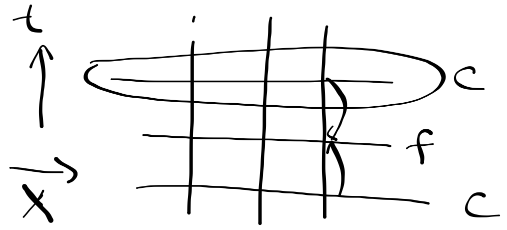








$u_t - u_{xx} = f$



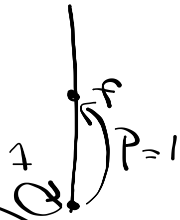
BE
$$\begin{bmatrix} 0 & 0 & 0 \\ -\alpha & 2\alpha+1 & -\alpha \\ 0 & -1 & 0 \end{bmatrix}$$

$\alpha = \frac{\Delta t}{\Delta x^2}$

FE
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ -\alpha & 2\alpha-1 & -\alpha \end{bmatrix}$$

$CN = \frac{1}{2}(BE + FE)$

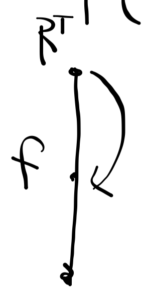
$P_i = -A_{ff}^{-1} A_{fc}$



$A_c = \underline{\underline{P^T A P}}$

BE → BE

FE → CN



BE → CN → $A_c = RAP$

BE → BE → $A_c = RAR^T$

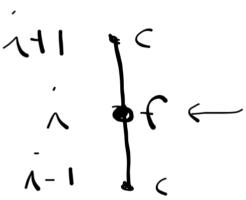
RAP

FE	P	$P = R^T$
$R = P^T$	CN	
R	(FE)	CN

BE → CN

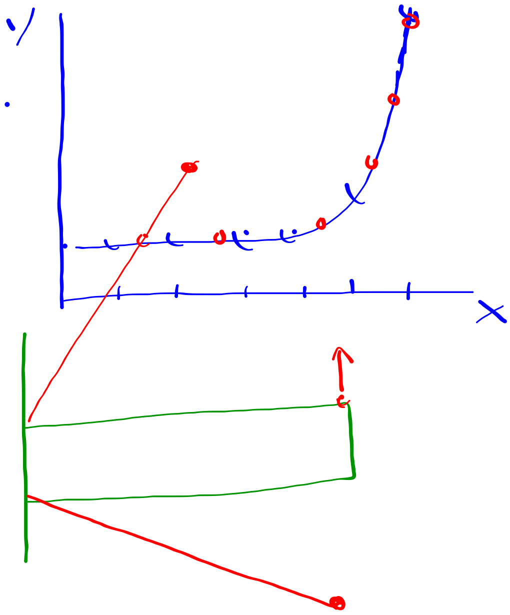
CN →

[ ]



$a e_{i+1} + c e_i + b e_{i-1} = 0$

$e_i = -\frac{a}{c} e_{i+1} - \frac{b}{c} e_{i-1}$



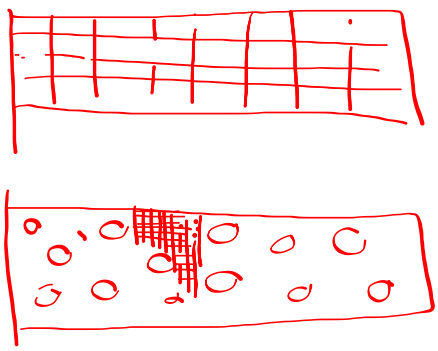
$$A = \begin{bmatrix} \text{Physical} \\ \text{mesh} \end{bmatrix}$$

$$F^h(u^h) = 0$$

$$F^h(u_0^h + \delta_0^h) = 0$$

32h "u_0^h"

$$F^{64h}(``u_0^h" + \delta_0^{64h}) = 0$$



$$F^0(u^0)$$

$$F^h(u^h)$$

$\tilde{e}_1^h$

A jacket

$$\frac{F^h(u^h + P^{2h}u^{2h})}{F^{2h}(u^{2h})}$$

FAS

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \cdot \qquad c_1\|\bar u\|_1^2 \le F(\bar u) \le c_2\|\bar u\|_1^2$$

$$P = \begin{bmatrix} P_{11} & 0 \\ 0 & P_{22} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1-\epsilon \\ 1+\epsilon & 1 \end{bmatrix}\begin{bmatrix} -\partial_{xx} & 0 \\ 0 & -\partial_{yy} \end{bmatrix}\begin{bmatrix} 1 & 1+\epsilon \\ 1-\epsilon & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1-\epsilon \\ 1+\epsilon & 1 \end{bmatrix}\begin{bmatrix} -\partial_{xx} & -\partial_{xx}-\epsilon\partial_{xx} \\ -\partial_{yy}+\epsilon\partial_{yy} & -\partial_{yy} \end{bmatrix}$$

$$= \begin{bmatrix} -\partial_{xx}-\partial_{yy}+\epsilon\partial_{yy} & -\partial_{xx}-\dots-\partial_{yy} \\ & -\partial_{xx}-\partial_{yy} - \epsilon\partial_{xx} \end{bmatrix}$$

FosLS

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ & A_{22} & A_{23} \\ & & A_{33} \end{bmatrix}\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \qquad u_1 \in H^1 \quad \begin{pmatrix} u_2 \\ u_3 \end{pmatrix} H(\mathrm{Div})$$

$$c_1\left(\|u_1\|_1 + \left\|\begin{matrix} u_2 \\ u_3 \end{matrix}\right\|_{Div}\right) \le F(u) \le C_2\|\quad\|$$

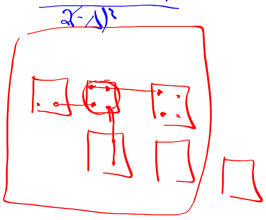
$$\frac{\sqrt{\frac{C_2}{C_1}} - 1}{\sqrt{\frac{C_2}{C_1}} + 1} \qquad A_{ii} \sim -\Delta$$

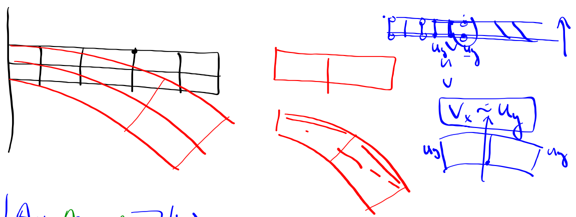
$$\frac{\left\langle \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & \boxed{A_{22}} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}, \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \right\rangle}{\left\langle \begin{bmatrix} A_{11} & 0 & 0 \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix}\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}, \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \right\rangle} \le C_1$$

$$\begin{bmatrix} -2\Delta & 0 & \partial_{xx}-\partial_{yy} \\ 0 & -\Delta & \partial_{xy} \\ \partial_{xx}-\partial_{yy} & \partial_{xy} & -\Delta \end{bmatrix} = \begin{bmatrix} -2\Delta & 0 & 0 \\ 0 & -\Delta & 0 \\ \partial_{xx}-\partial_{yy} & \partial_{xy} & \Delta \end{bmatrix}\begin{bmatrix} (-2\Delta)^{-1} & & \\ & (-\Delta)^{-1} & \\ & & (-\frac{1}{2}\Delta)^{-1} \end{bmatrix} ''$$

$$C_1 = \frac{1}{D^3}, \quad C_2 \sim O(1)$$

$$\frac{(\partial_{xx}+\partial_{yy})^2 + \partial_{xy}^2}{2(-\Delta)^2}$$





$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ \cdot & A_{22} & \cdot \\ \cdot & \cdot & A_{33} \end{bmatrix}\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \approx \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

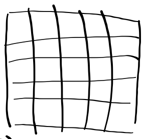
Dirac eq.

$$\boxed{\vec{b}\cdot\nabla u = f}^{+(-\varepsilon\Delta u)}$$

$$\sum_{\mu=1,2} \gamma_\mu \underbrace{(\partial_\mu + iA_\mu)}_{\nabla_\mu} \vec{\Psi} + mI\vec{\Psi} = \vec{f}$$

$$\gamma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \vec{A} = (A_1, A_2)$$

$$\gamma_2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$



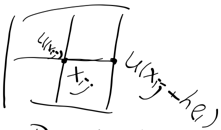
Wilson

$$\sum_\mu h(\partial_\mu + iA_\mu)(\partial_\mu - iA_\mu)\vec{\Psi}$$

$$D_w \vec{\Psi} = \vec{f}$$

$$(D_w\vec{\Psi}, \underset{\substack{\cap \\ \text{Range}(D_w)}}{\vec{v}}) = (\vec{f}, \vec{v}) \quad \text{Is there ?}$$

$$\nabla_\mu u = \frac{e^{ihA_\mu} u(x_{ij} + he_\mu) - u(x_{ij})}{h}$$



$$\nabla_\mu^* u = \frac{u(x_{ij}) - e^{-ihA_\mu}(x_{ij} - he_\mu)}{h}$$

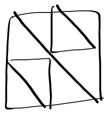
Dirac-Wilson

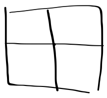
$$\frac{1}{2}(\nabla_\mu + \nabla_\mu^*)\vec{\Psi} - h\nabla_\mu\nabla_\mu^*\vec{\Psi} + mI\vec{\Psi} \approx D_w\vec{\Psi}$$

Background.

$$A_{ij} = (\nabla\phi_i, \nabla\phi_j)$$

FD, FE





Back to Dirac

$$\vec{\Psi} \in (Q_1)^2$$

EAFE

$$\nabla\cdot(\underbrace{\nabla u + \vec{\beta}u}_{\text{approximated by a const}})$$

test $\vec{v} \in ND(\mathbb{C}) \cdot \overbrace{e^{ihA_\mu}}^{\text{on each edge}}$

assume $\nabla_\mu \vec{\Psi}$ is approx. by a const vec on each elem

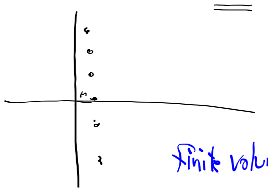
Dirac term's dis $\frac{1}{2}(\nabla_\mu + \nabla_\mu^*)\vec{\Psi}(x_{ij})$

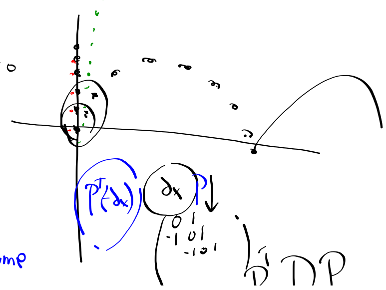
$$A_{ij} = (Au, v) \quad \approx \sum_{\mu=1,2} (\gamma_\mu \nabla_\mu \vec{\Psi}, \vec{v})$$

$$Du = f$$

$$(Du, \underset{\substack{\cap \\ R(D)}}{v}) \geq \|u\|_D^2$$

$$(D + mI)_{+-h\Delta} \quad m=0$$





finite volume

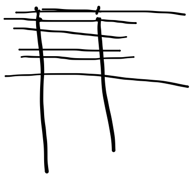
PDE-Based → bad coercivity

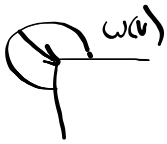
→ Fix system AMG → unweighting

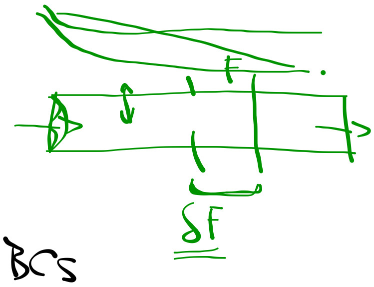
→ αAMG h^{-1} → wrongly weighted BCs

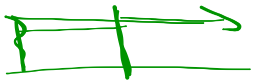
→ αSA

→ SA









$\|L(u)\|^2$

① Gauss Newton $\|L'(u)v\|^2$ convex

② Newton-Raphson $\mathcal{L}'^* L(u)=0$

$J(u) = \langle \mathcal{L}(u) - f, \mathcal{L}(u) - f \rangle$ $\mathcal{L}(u)$ quadratic Quarticu

$\langle \mathcal{L}(u) - f, \mathcal{L}'(u) v \rangle = 0$

u_0

$\langle \mathcal{L}'(u_0)\delta u - (f - \mathcal{L}u_0), \mathcal{L}'(u_0)\delta u - (f - \mathcal{L}(u_0)) \rangle$

$u_0 + \alpha \delta u$

q^H u^H $\langle \mathcal{L}(u^H) - f, \mathcal{L}(u^H) - f \rangle_\tau$

$\varepsilon_1 > \varepsilon_2 > \varepsilon_3 \cdots \varepsilon_{35} \cdots$

Acc/cost

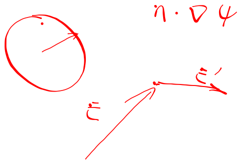
$F(u)$ $F(u + \delta u) \approx$

$F(u_0) + F'(u_0)\delta u + F''(\tilde{u}_0)[\delta u, \delta u]$

$\frac{\partial \psi}{\partial t}$

$\Omega \cdot \nabla \psi + \sigma_t \psi = \sigma_s \phi$ (E, x, t) $\sigma_a = \sigma_t - \sigma_s$

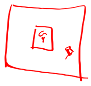
$n \cdot \nabla \psi = g$ inflow



$\psi(x, \Omega, E, t)$

$\phi = \int \psi(x, \Omega, E, t)\, d\Omega$

finite element (discontinuous Galerkin)



discretized in $\Omega \to$ piecewise constants S_n

$\Omega \to$ spectral in $\Omega \to P_n$

$$\left[\quad\right]\begin{pmatrix} \psi_1(x, \Omega_1, E, t) \\ \psi_2(x, \Omega_2, E, t) \\ \vdots \\ \psi_n(x, \Omega_n, E, t) \end{pmatrix} \qquad \psi(x, \Omega, E, t) \approx \sum_{i=1}^{n} \psi(x, \Omega_i, E, t)$$

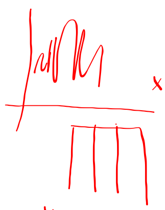
$$\begin{pmatrix} H_1 & & & -I\sigma_s \\ & H_2 & & -I\sigma_s \\ & & \ddots & \vdots \\ & & H_n & -I \\ \delta I & \delta I & \cdots \delta I & \end{pmatrix}\begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_n \\ \phi \end{pmatrix} = f \qquad \frac{\sigma_t}{\sigma_s} \approx 1$$

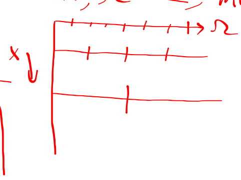
$H_i = \Omega_i \cdot \nabla + \sigma_t$

$S\, k(x, x')\phi(x')\,dx'$

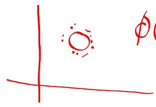
$\sum_{i=1}^{n} [\Omega_i \cdot \nabla + \sigma_t]^{-1}$

$x, \Omega \Rightarrow$ multigrid space-angle



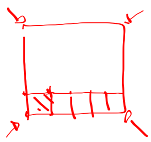


near null space



$\phi(x)$

$\rho^{1/2} + N$



$\mathcal{F}(\phi_1, \phi_2) = \|\Omega \cdot \nabla \quad\|^2$

σ_t, σ_s
nodal

$\Omega \cdot \nabla \psi + \sigma_t \psi = S\psi + q$

$L\psi - S\psi = q$

$\langle L\psi^h - S\psi^h, Lv^h \rangle = \langle q, v^h \rangle$

$\langle L\psi^h, Lv^h \rangle = \langle S\psi^h, Lv^h \rangle + \langle q, v^h \rangle$

$\langle (\Omega \cdot \nabla + \sigma_t)\psi^h, (\Omega \cdot \nabla + \sigma_t) v^h \rangle$

$\langle \Omega \cdot \nabla \psi^h, \Omega \cdot \nabla v^h \rangle$

$-\nabla \cdot (\Omega \Omega^T) \nabla \psi$

$\sigma = .3 \;.5$

$\rho = .56$

$(-1, 3, -1) \otimes$