$\phi(x^{(k)}) \longrightarrow x^{(k+1)}, \quad x^{(0)} = g$

$$\begin{bmatrix} I & & & \\ -\phi & I & & \\ & -\phi & I & \\ & & \ddots & \ddots \end{bmatrix} \begin{bmatrix} x^0 \\ x^1 \\ x^2 \\ \vdots \end{bmatrix} = \begin{bmatrix} g \\ 0 \\ 0 \\ \vdots \end{bmatrix}$$

$$\begin{bmatrix} I & & & \\ -\phi_\Delta & I & & \\ & -\phi_\Delta & I & \\ & & \ddots & \ddots \end{bmatrix}$$

$\phi^2 \simeq \phi_\Delta$

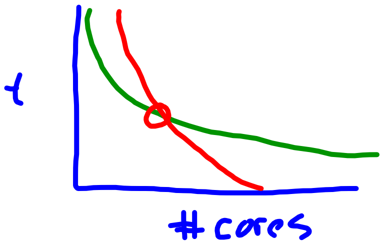
XBraid

$$P = \begin{bmatrix} I & & \\ -\phi & & \\ & I & \\ & -\phi & \\ & & \ddots \end{bmatrix}$$

Sep 13-8:09 AM

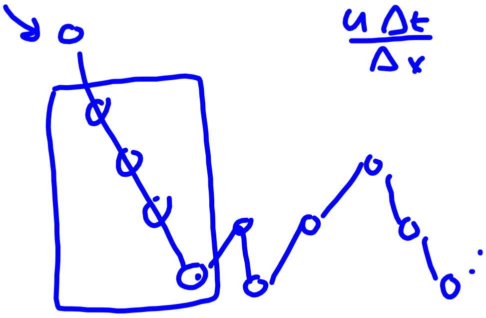
$$\phi = (I + \Delta t A)^{-1}$$

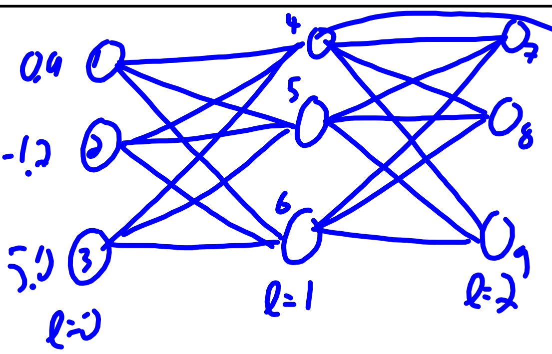
$$\phi = (I - \Delta t A)$$



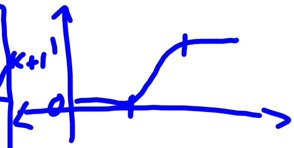
$$u_t + u u_x = 0$$

$$\frac{u \Delta t}{\Delta x}$$





$$\phi_k\left(\left[\underline{W}\right]^k\right) \to \left[\underline{W}\right]^{k+1}$$



$$f(0.9w_1 + (-1.2)w_2 + 5w_3) = n_4$$

$$\begin{bmatrix} n_7 \\ n_8 \\ n_9 \end{bmatrix} - \begin{bmatrix} y_0 \\ y_1 \\ y_2 \end{bmatrix} \quad \begin{matrix} 1 \\ 0 \\ 0 \end{matrix}$$

Sep 13-9:07 AM

$$\begin{bmatrix} I & & & & & \\ -\phi_0 & I & & & & \\ & -\phi_1 & I & & & \\ & & -\phi_0 & I & & \\ & & & -\phi_1 & I & \\ & & & & \ddots & \ddots \end{bmatrix} \begin{bmatrix} w^{(0)} \\ w^{(1)} \\ w^{(2)} \\ \vdots \end{bmatrix} = \begin{bmatrix} g \\ 0 \\ 0 \\ \vdots \end{bmatrix}$$

$$w + \alpha \hat{\phi}(w)$$

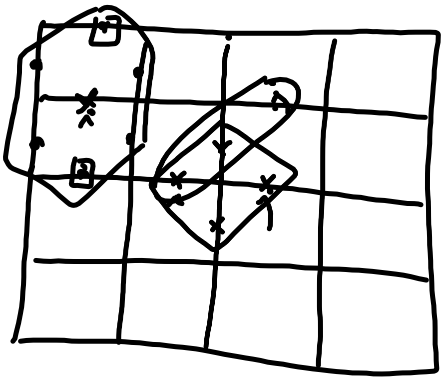
$$\Sigma$$

$$\|$$

$$n_j = W_{ji} \; n_i$$

$$f(n_j)$$

Automatic Construction of smoothers in AMG

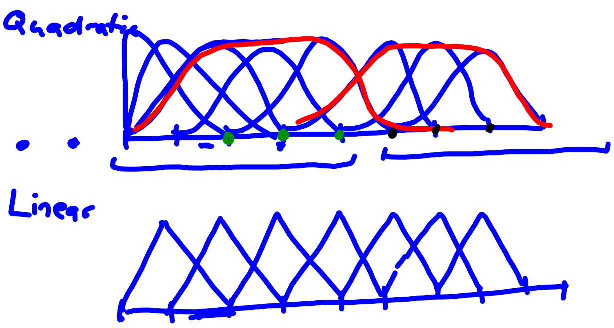


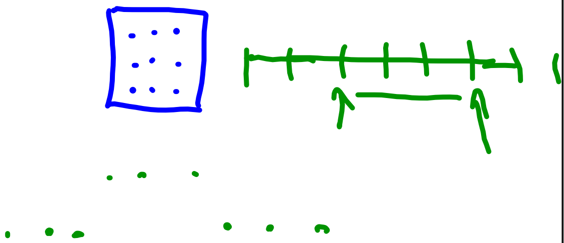
$$\nabla \times \nabla \times u + \sigma u = f$$

$$(I - G(G^T A G)^{-1} G^T A)$$

$$\Omega_i \to A_i$$

Smoothed Aggregation w/ IGA

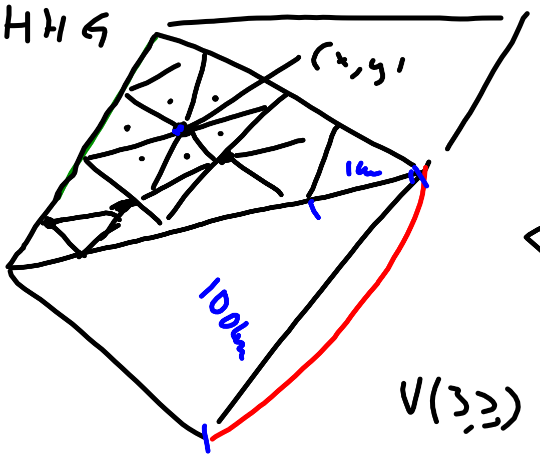




$$u'''' = f$$
$$u(0) = u'(0) = 0$$

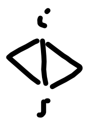
$$\Rightarrow \boxed{\begin{aligned} v - u'' &= 0 \\ v'' &= f \end{aligned}}$$

HHG



$$\nabla \kappa(x) \nabla u = f$$

$$A = (a_{ij})_{i,j=1,\dots,N}$$

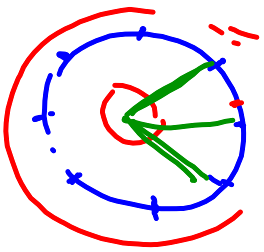


$$a_{ij} = \int \underbrace{\kappa(x)}_{\text{polynomial}} \nabla \varphi_j \nabla \varphi_i \, dx$$

$\triangleleft \quad \triangleright$

$$a_{ij}(x,y) = p_j(x,y) \approx \hat{p}_j(x,y) \in P^2$$

$\boxed{x}$



$$\Delta u + k^2 u = f$$

$$v_1, v_2, \ldots, v_k$$

$$e = \sum a_j v_j$$

$$e^{ik_1 x + ik_2 y}, \quad (k_1, k_2)$$

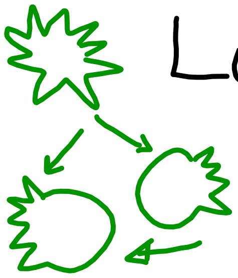
$$e = \sum P_j a_j^c$$

$$e(\vec{x}) = \sum a_i(\vec{x})\, e^{ik_i \vec{x}}$$

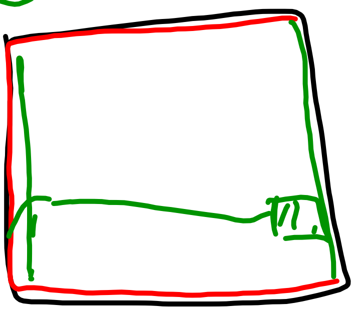
$$Le(x) = \sum e^{ik_i x} L_i a_i(\vec{x})$$

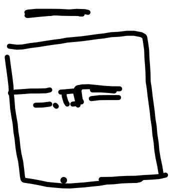
$$\left[P_1^* A P_1 \quad P_1^* A P_2 \ldots P_1^* A P_J \right.$$

NIRD

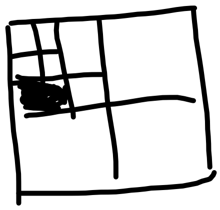


$Lu = f$





- $Lu_0 = f \quad \approx P$
- $\Omega = \bigcup_\ell \Omega_\ell$

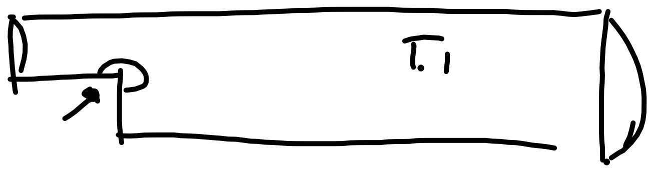


- Iterate:

  $\rightarrow$ Solve $L\delta u_\ell = \chi_\ell r$

  $u \leftarrow u_0 + \sum_\ell \delta u_\ell$

Sep 14-12:07 PM



Sep 14-12:37 PM

$(P^{\frac{1}{2}} + m)$ $\boxed{\Omega_{x_j} \cdot \nabla \psi_{e_j} + \sigma_t \psi_{e_j}}$

$\underline{L^{*}}(\underline{L}\psi = q)$

$L\psi = q$

I- F

I- L

$$\begin{bmatrix} A_{ff} & A_{fc} \\ A_{cf} & A_{cc} \end{bmatrix}$$

XAIR

$$[-A_{fc}\Delta \quad I] \begin{bmatrix} A_{ff} & A_{fc} \\ A_{cf} & A_{cc} \end{bmatrix} \begin{bmatrix} W_0 \\ I \end{bmatrix}$$

$$A_{ff} = I - L_{ff}$$

$$A_{ff} = \sum_{j=0}^{K} L_{ff}^{j}$$

$$\Delta = \sum_{j=0}^{\ell} L_{ff}^{j}$$

$$[0 \;,\; A_{cc} - A_{cf} A_{ff}^{-1} A_{fc}]$$

Sep 14-1:29 PM

$$\left(\begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} - \boxed{\begin{bmatrix} \underline{W_0} \\ I \end{bmatrix}} \left[[z_q, I] \begin{pmatrix} A_{ff} & A_{fc} \\ A_{cf} & A_{cc} \end{pmatrix} \begin{bmatrix} \underline{W_0} \\ I \end{bmatrix} \right)^{-1} (z, I) \begin{pmatrix} A_{ff} & A_{fc} \\ A_{cf} & A_{cc} \end{pmatrix} \right)$$

F-point Relax

$$\boxed{\begin{matrix} I - \Delta A_{ff} & -\Delta A_{fc} \\ 0 & I \end{matrix}} \begin{vmatrix} \hat{W} \\ I \end{vmatrix} = \begin{vmatrix} (I - \Delta A_{ff}) W - \Delta A_{fc} \\ I \end{vmatrix}$$

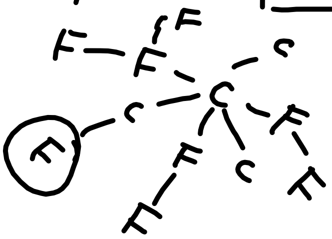
$$\begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} - \underbrace{\begin{pmatrix} I & \hat{W} \\ 0 & I \end{pmatrix} \begin{pmatrix} \Delta & 0 \\ 0 & K^{-1} \end{pmatrix} \begin{pmatrix} I & 0 \\ z & I \end{pmatrix}}_{} \begin{pmatrix} & A & \end{pmatrix}$$

$$M = \begin{pmatrix} I & 0 \\ -z & I \end{pmatrix} \begin{pmatrix} \Delta^{-1} & 0 \\ 0 & K \end{pmatrix} \begin{pmatrix} I & -\hat{W} \\ 0 & I \end{pmatrix}$$

Sep 14-1:37 PM

Hypre AIR

1. (L)AIR-1 : restriction op
2. AIR-2
3. 1-pt P
4. C-only , F-only



R: $\left[\, \times \times \times \times \;\middle|\; I \,\right]$

A_{FF}

A_{CF}

$\left[\, -A_{CF} A_{FF}^{-1} \,\right]$

$$-\Delta u + \vec{a}^T \cdot \nabla u = f$$

$$150/(2\cdot 64) = 1.17$$

$$\vec{a} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} = \begin{pmatrix} 150 \\ 150 \\ 150 \end{pmatrix}$$

64^3, V-cycles, (0,2F), HMIS

STD: 0.37, grid complex 1.8, op complex 5.3, cycle 10.6

AIR-1: 0.28 " 1.6, " 2.7 " 1.9

AIR-2: 0.33 " 1.6, " 3.1 " 2.0

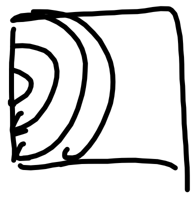
$$-\varepsilon \Delta u + a u_x + b u_y = f$$

$$\frac{1}{h^2}\begin{bmatrix} & -\varepsilon + bh\mu_y & \\ -\varepsilon + ah(\mu_x - 1) & -\Sigma & -\varepsilon + ah\mu_x \\ & -\varepsilon + bh(\mu_y - 1) & \end{bmatrix}$$

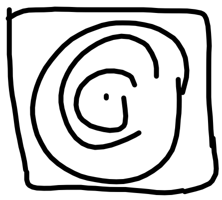
$$\mu_x = \begin{cases} \varepsilon/2ah & , ah > \varepsilon \\ 1 + \varepsilon/2ah & , ah < -\varepsilon \\ 1/2 & , |ah| < \varepsilon \end{cases} , \quad \mu_y = \Big\{ \quad a \leftrightarrow b$$

1) $a(x,y) = \sin(\frac{\pi}{8}$, $b = \cos(\frac{\pi}{8}$, $l=0,1,\dots 4$

2) $a(x,y) = (2y-1)(1-x^2)$, $b = 2xy(y-1)$.



3) $a(x,y) = 4x(x-1)(1-2y)$, $b = -4y(y-1)(1-2x)$



$\varepsilon = 10^{-6}$

Sep 14-2:48 PM

STD-AMG | AIR-1 (-2)

N	NP	CF	#iter	T-Setup	T-Sol	CF	#iter	T-setup	T-sol
512^2	1	.26	18	.29	.60	.24	17 (13)	.29	0.48
1024^2	4	.26	18	.36	.71	.24	17 (16)	.34	0.58
2048^2	16	.36	24	.62	1.17	.37	25 (23)	.40	1.06
4096^2	64	.37	25	.83	1.32	.45	43 (36)	.40	1.85
8192^2	256	.49	27	.91	2.10	—	>100 (>100)	.52	—

$\varepsilon = 10^{-6}$

(0, 2F)

(C0, FFC,

$Q = VU^*$

$L = U\Sigma V^*$ $\quad \sqrt{L^*L} = V\Sigma V^* = QL$

$L^*L\,u = L^*f$ $\quad \sqrt{LL^*} = U\Sigma U^* = LQ$

${}_N\boxed{P}_{n_c}\; R$ $\quad L_c = R^*LP$

$$\underbrace{P(R^*LP)^{-1}R^*L}_{\Pi}\underline{e}$$

QL orth proj onto $R(P)$

$\Pi_1 = P(P^*QLP)^{-1}P^*QL$

$\Pi_2 = R(R^*LQR)^{-1}R^*LQ$

$(QR)\left[(QR)^*\underline{QL}(QR)\right]^{-1}(QR)^*Q\underline{L}$

SAP

SAP

P sat SAP on A w/ const K

$$\min_{u_c} \|v - Pu_c\|_A^2 \le K \|Av\|^2$$

SSAP

$$\min_{u_c} \|v - Pu_c\|^2 \le K \|Av\|^2$$

WAP

$$\min_{u_c} \|v - Pu_c\|^2 \le K \|v\|_A^2$$

$$\underset{K}{\text{SAP}} \Longleftrightarrow \underset{K^2}{\text{SSAP}} \qquad \underset{K}{\text{SAP}} \Rightarrow \underset{K}{\text{WAP}}$$

GS Thesis

$\|\Pi\|_{QL} \leq C$ P has SAP wrt QL_K

V^2 relax $v(k) \Rightarrow$ convergence

P approx V

R approx U

$$P_{n_c} = \begin{bmatrix} v_1, & \cdots & v_{n_c} \end{bmatrix} \qquad R = \begin{bmatrix} u_1 & \cdots & u_{n_c-1} & u_{n_c+1} \end{bmatrix}$$

$$QL = V\Sigma V^* \qquad LQ = U\Sigma U^*$$

$$R^* L P$$

P SAP w/ QL K

$$\Pi = P(R^{*} L P)^{-1} R^{*} L$$

$$(PB)([R^{c}]^{*} L P B)^{-1} (R^{c})^{*} L$$

$$P \to \left[w_1 \ldots w_{n_c} \right]$$

$$w_j = \Pi_1 v_j$$

$$w_j = v_j - n_j \quad \|n_j\|_A^2 \le K \sigma_j^2$$

$$\left[w_1 \ldots w_\ell \,\middle|\, - \right]$$

$$\sigma_j K < 1/3$$

$$j < \ell$$

$$R = \left[\begin{array}{c|c} z_1 \cdots z_\ell & Z_2 \end{array}\right]$$

$z_i = \Pi_2 u_i \quad i = 1 \ldots \ell$

$z_i = u_i - m_i \quad \|m_i\|^2 \le K\sigma_i^2$

$$U^* R = \left[\begin{array}{c|c} I & 0 \\ \hline 0 & Z_3 \end{array}\right] + \left[\begin{array}{c|c} M & 0 \end{array}\right]$$

$$R^* L P$$

$$\left\| \boxed{Z_2^* L W_2}^{-1} \right\| \leq \frac{1}{\delta}$$

$$(V^* R)^* \Sigma (V^* P)$$

$$\begin{bmatrix} I_k & \\ Z_1 & Z_2 \end{bmatrix} \qquad \begin{bmatrix} I_k & \\ W_1 & W_2 \end{bmatrix}$$

V-cycle Proof

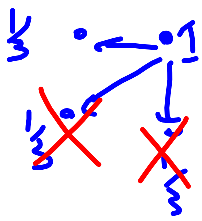
P SAP $\quad \Pi$ stable $\Rightarrow$ R WAP

R $\quad \Pi$ Stable $\Rightarrow$ P WAP

$$A_{ff} \quad P\left[R^{x} L(P+\beta)\right]^{-1} \qquad P = \begin{pmatrix} W \\ I \end{pmatrix}$$

$$I - L_{ff} \quad L^{k}_{ff}$$

$$\begin{bmatrix} (1 - \Delta A_{ff})W + -\Delta_f A_{fc} \\ I \end{bmatrix}$$



$\left(\frac{2}{3}\right)^3$ $\frac{8}{27}$

2

L^{3}_{ff}

$$\begin{bmatrix} N^T K^{-1} N + D & 0 \\ T^T K^{-1} N + G_m & T^T K^{-1} T + H \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} +$$

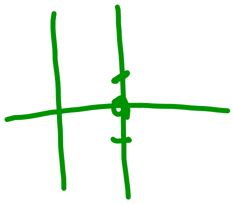
$$\begin{bmatrix} N^T K^{-1} N + D & N^T K^{-1} T + \frac{G_m^*}{2} \\ T^T K^{-1} N + \frac{G_m}{2} & T^T K^{-1} T + H \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = rhs$$

$$= \begin{bmatrix} N & \\ & T \end{bmatrix}^T \begin{bmatrix} I \\ I \end{bmatrix} K^{-1} \begin{bmatrix} I & I \end{bmatrix} \begin{bmatrix} N & \\ & T \end{bmatrix} + \begin{bmatrix} D & 0 \\ G_m & H \end{bmatrix}$$

$$A = \frac{(A + A^T)}{2} + \frac{(A - A^T)}{2} =$$

$$\begin{pmatrix} 0 & -\frac{G_m^*}{2} \\ \frac{G_m}{2} & 0 \end{pmatrix} = A_N$$

$$\frac{A_S + A_N}{I + A_S A_N}$$



$$\begin{pmatrix} N & 0 \\ 0 & T \end{pmatrix}^T \begin{pmatrix} I \\ I \end{pmatrix} K^{-1} (I\ I) \begin{pmatrix} N & 0 \\ 0 & T \end{pmatrix} =$$

$$L^{-1}\left[\underbrace{\begin{pmatrix} N \\ T \end{pmatrix}^T}_{U} \underbrace{K^{-1}}_{C} \underbrace{(N\ T)L^{-1}}_{V} + I\right]^{-1}$$

$$= L^{-1} - L^{-1}$$

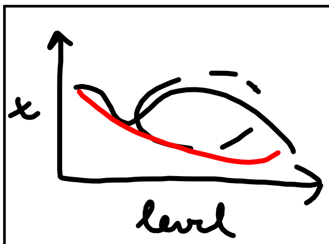
$$(A + UCV)^{-1} \quad (I + UCV)^{-1}$$

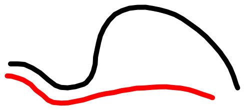
$$= A^{-1} - A^{-1}U\left(C + (VA^{-1}U)^{-1}\right)VA^{-1}$$

$$(I + UCV)^{-1}$$

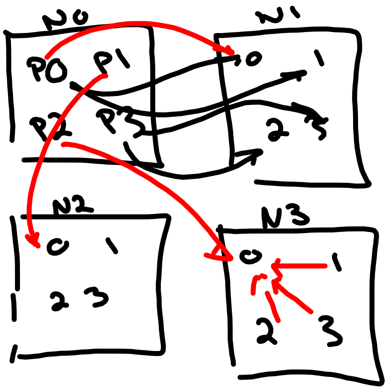
$$= I - U\left(C + (VU)^{-1}\right)V$$

$$= I - \begin{pmatrix} N \\ T \end{pmatrix}^T \left(K^{-1} + \left((N\ T)L^{-1}\begin{pmatrix} N \\ T \end{pmatrix}\right)^{-1}\right)(N\ T)L^{-1}$$





$\alpha n + \beta s$



$$\begin{bmatrix} A_{11} & \underline{A_{12}} \\ \underline{A_{21}} & A_{22} \end{bmatrix}, \quad \begin{bmatrix} -\underline{\Lambda} & \\ & -\underline{\Lambda} \end{bmatrix},$$

$H \rightarrow H'$ $\qquad$ $H' \rightarrow H$

$H' \rightarrow H'^{-1}$

$$\begin{bmatrix} H & L \\ L^T & 0 \end{bmatrix} = \begin{bmatrix} I & \\ L^T H^{-1} & I \end{bmatrix} \begin{bmatrix} H & \\ & S \end{bmatrix} \begin{bmatrix} I & H^{-1}L \\ & I \end{bmatrix}$$

$$S = -L^T H^{-1} L$$

$$\text{Inv} \rightarrow \begin{bmatrix} I & -H^{-1}L \\ & I \end{bmatrix} \begin{bmatrix} H^{-1} & \\ & S^{-1} \end{bmatrix} \begin{bmatrix} I & \\ -L^T H^{-1} & I \end{bmatrix}$$

$$(h^2 I)^{-1}$$

Sep 15-10:50 AM

$$L u = f \quad , L u = \nabla \cdot \underline{b} u + \sigma u$$

$$\min_{u^i} \| \hat{u} - u^i \|_0^2 \quad L^* u = \underline{b} \cdot \nabla u + \sigma u$$

$$\underline{u}^{i\in\phi_i} , \underline{z}^{i\in\psi_j}, \quad \pi ; L^2 \to L^*(z^{+1})$$

$$\min_{u^i} \| \pi(\hat{u} - u^i) \|_0^2$$

$$(H)_{ij} = (L^* \phi_j , L^* \psi_i) \quad (L)_{ij} = (\phi_j , L^* \psi_i)$$

$$\| \pi u^i \|_0 \geq C_I \| u^i \|_0 \quad C_I = O(h^{0.2})$$

$$C_I^2 x^T M x \leq - x^T \underset{\sim}{S} x \leq x^T M x$$

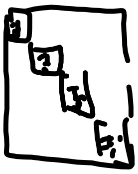
$\hat{Z}^H$

$(\hat{L}^T \hat{H}^{-1} \hat{L})^{-1}$

$\begin{bmatrix} \hat{H} & \hat{L} & -C \\ L^T & & \\ C^T & & \end{bmatrix} \begin{bmatrix} z \\ u \\ \mu \end{bmatrix}$

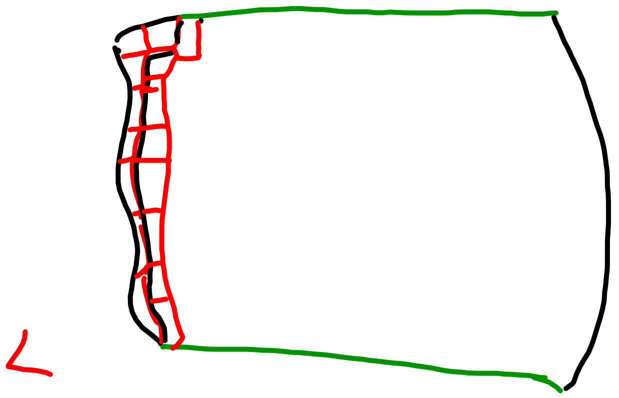
$B^{-1} \Lambda \mu = \hat{f} \quad B^{-1} S_x$

sparse

$\nabla \cdot$  $\nabla \sim$

$$\langle (-\Delta_{\frac{h}{2}})^{-1} \cdot\,, \cdot \rangle + \underbrace{h^2 \langle \cancel{\mathrm{div}\cdot}, \mathrm{div} \rangle}$$

$(-\Delta_{\frac{h}{2}})^{-1}$ $O(\varsigma^{0.2})$



G X=S

11 1 ?